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DEDS control synthesis problem solving

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Introduction

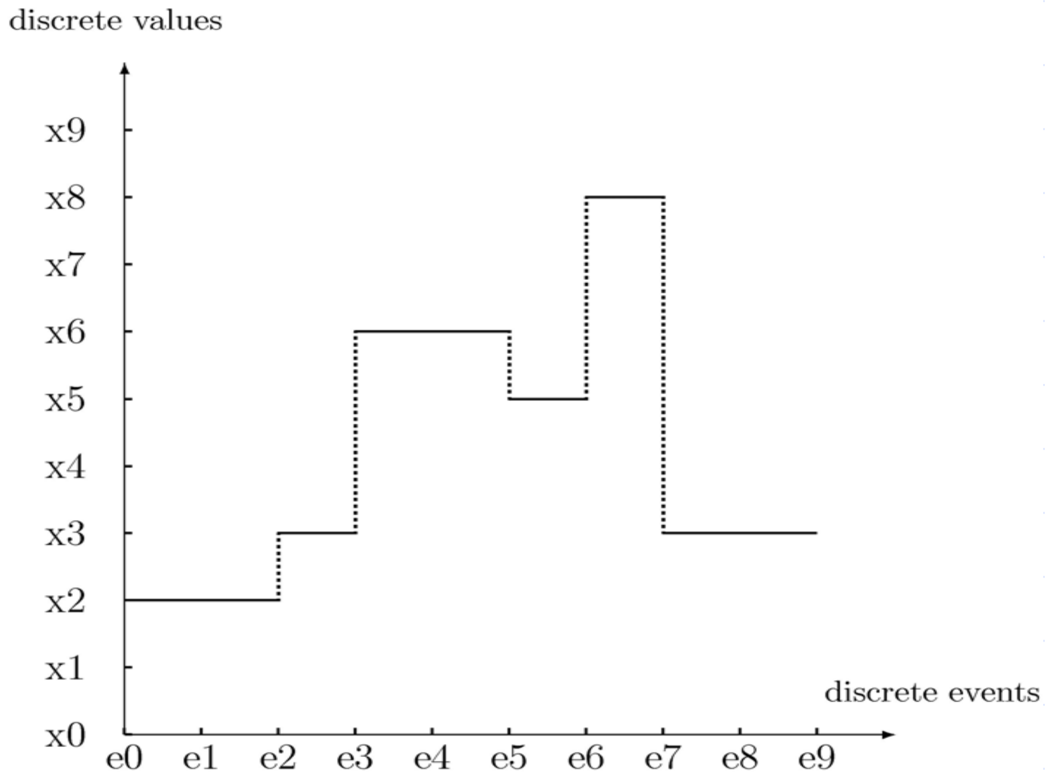
Basic definition of DEDS

DEDS (discrete event dynamic systems) are the systems where the development of the system dynamics depends on the occurency of discrete events, i.e. DEDS are systems driven by discrete events.

Typical kinds of DEDS

- flexible manufacturing systems
- communication systems
- transport systems

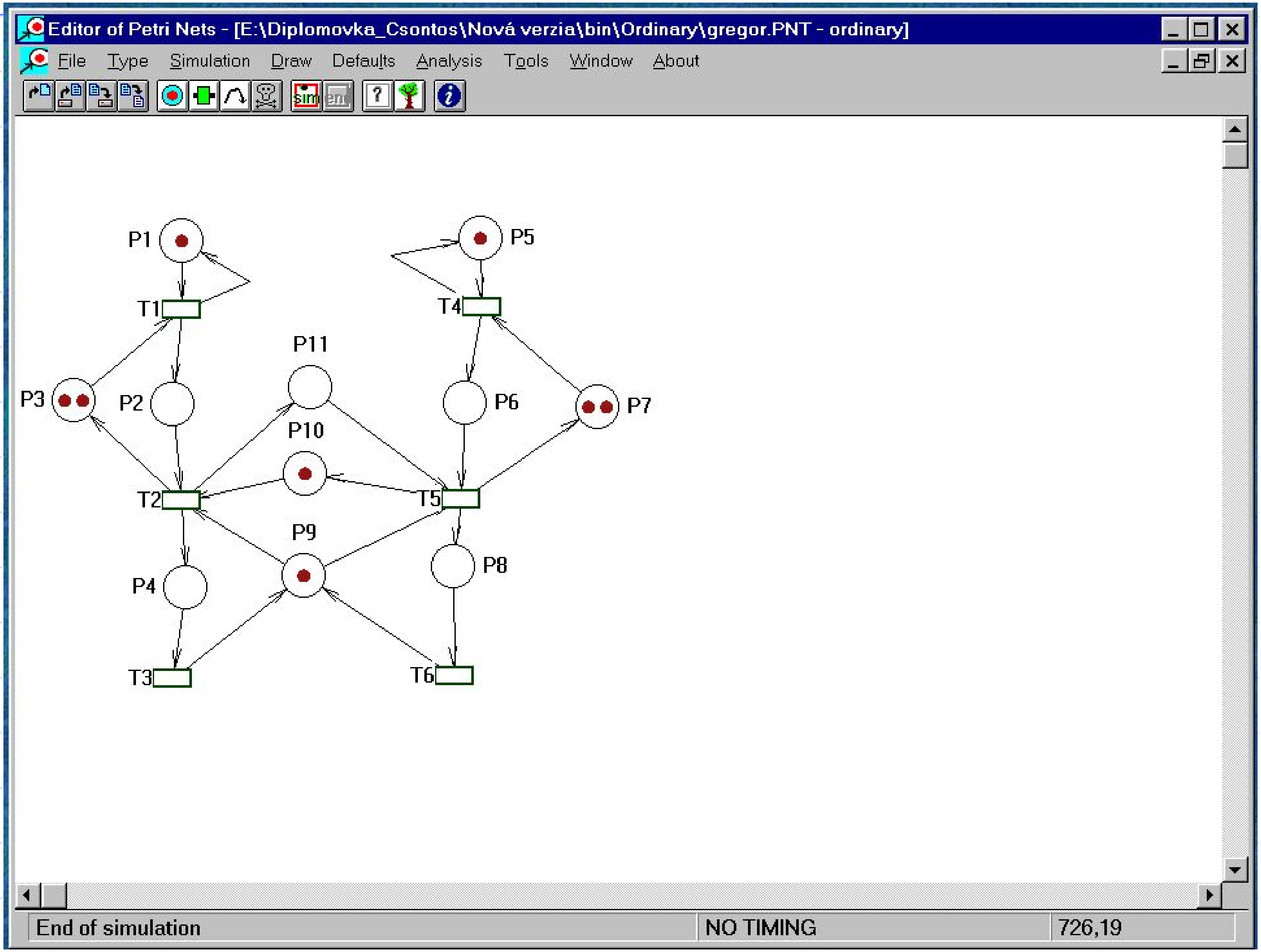
The graphical expression of a DEDS variable



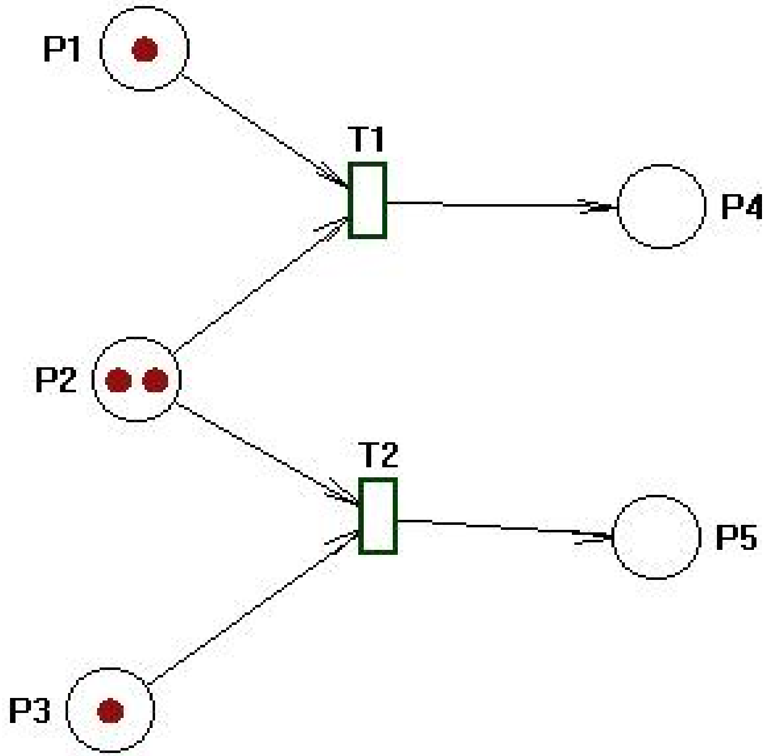
Petri nets (PN) in DEDS modelling

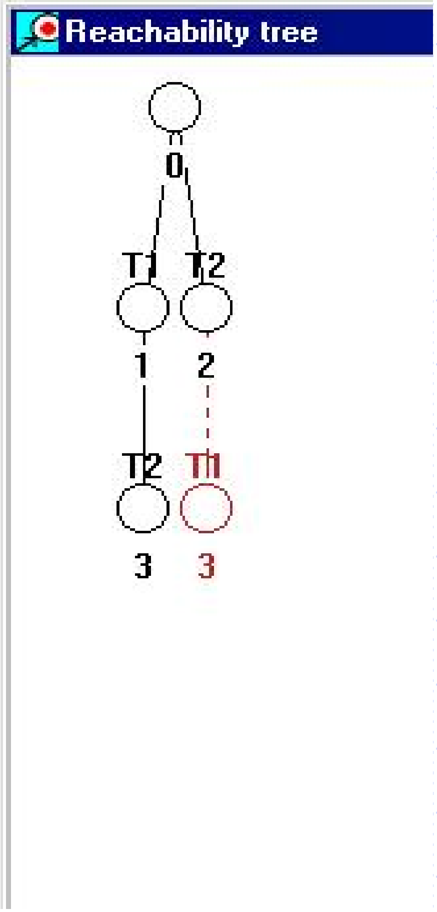
- PN are able to express parallelism and conflict situations
- PN can be expressed in analytical terms (in the form of the linear discrete system)
- PN properties can be tested by means of the reachability tree and invariants
- PN allow to use analytical approach to the DEDS control synthesis

Example of a Petri net

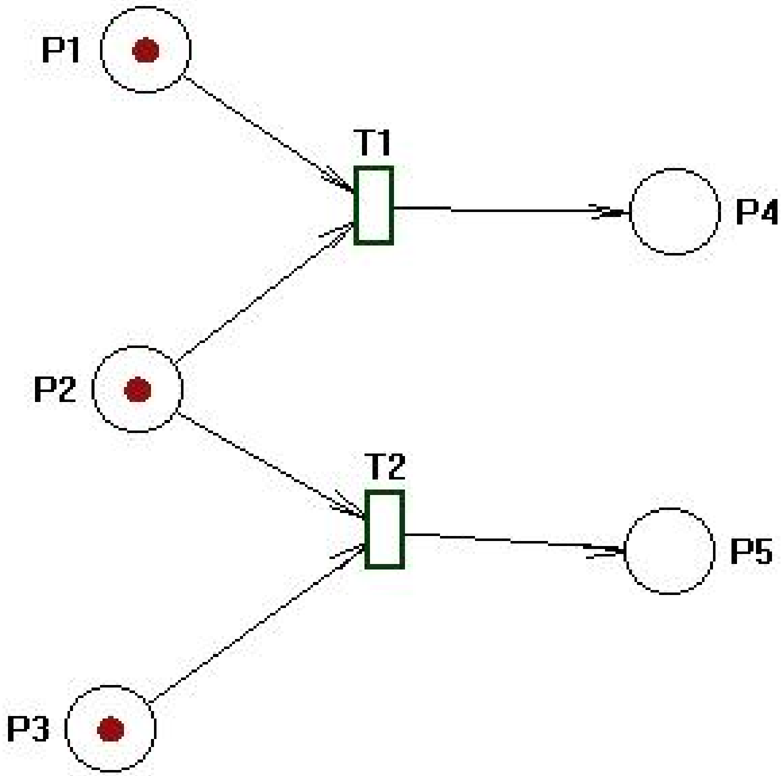


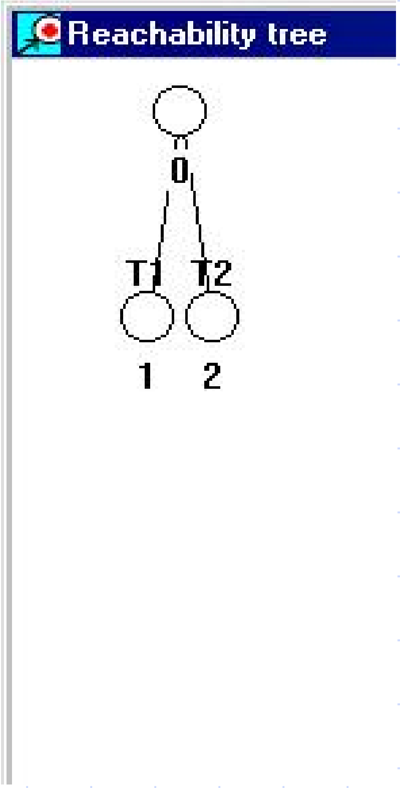
Parallelism





Conflict situation





Formal expression of the Petri net structure

$$\langle P, T, F, G \rangle; \quad P \cap T = \emptyset; \quad F \cap G = \emptyset$$

$$P = \{p_1, ..., p_n\}$$

$$T = \{t_1, ..., t_m\}$$

$$F \subseteq P \times T$$

$$G \subseteq T \times P$$

Formal expression of the Petri net dynamics

$$\langle X, U, \delta, \mathbf{x}_0 \rangle ; \quad X \cap U = \emptyset$$

$$X = \{\mathbf{x}_0, \ \mathbf{x}_1 \ldots, \ \mathbf{x}_N\}$$

$$U = \{\mathbf{u}_0, \ \mathbf{u}_1 \ \ldots, \ \mathbf{u}_N\}$$

$$\delta : X \times U \longrightarrow X$$

$\mathbf{x}_0$ is an initial state

Mathematical model of Petri net

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{B}.\mathbf{u}_k \quad , \quad k = 0, N$$

$$\mathbf{B} = \mathbf{G}^T - \mathbf{F}$$

$$\mathbf{F}.\mathbf{u}_k \leq \mathbf{x}_k$$

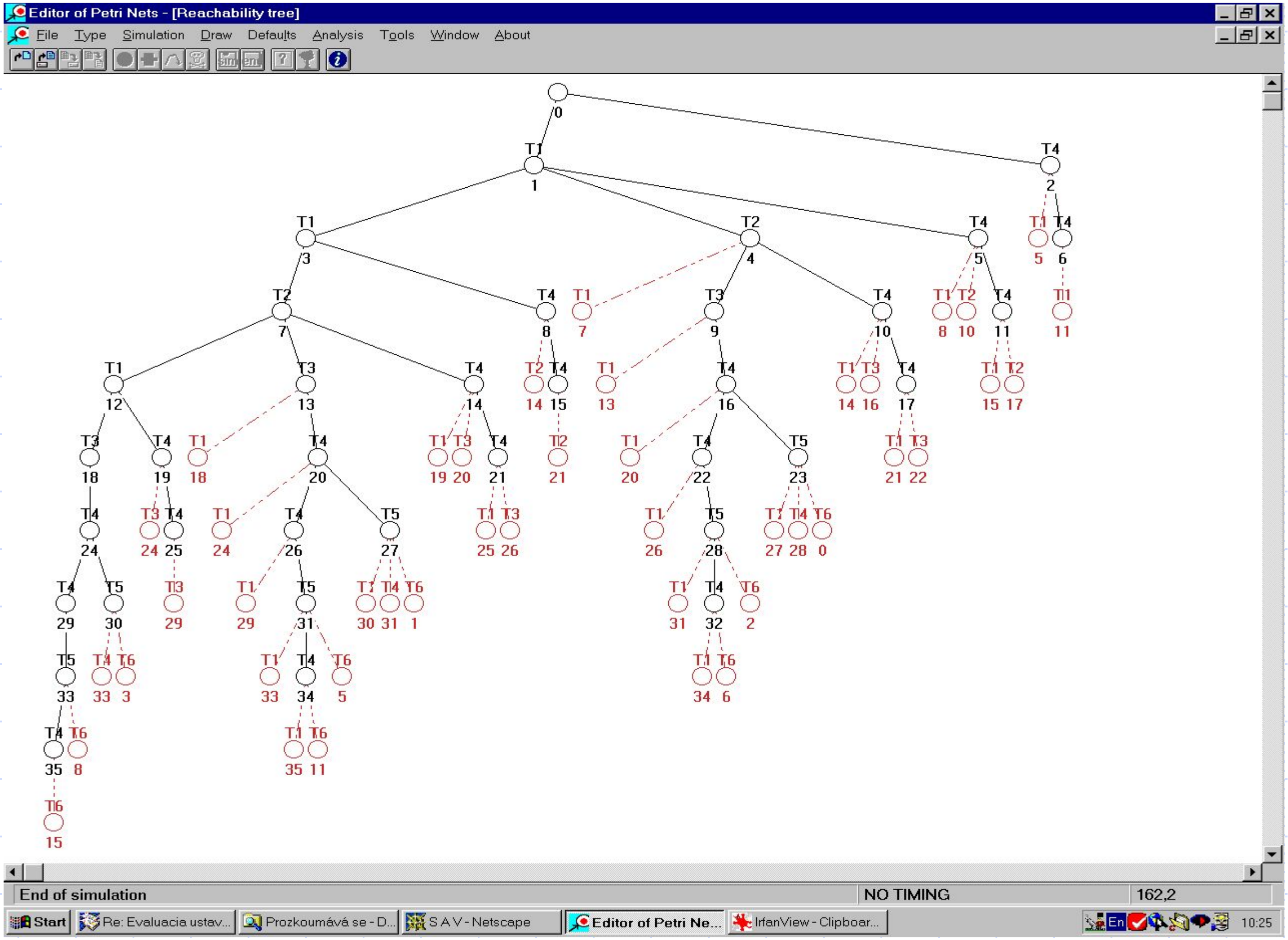
$$\mathbf{x}_k = (\sigma_{p_1}^k, \ldots, \sigma_{p_n}^k)^T$$

$$\sigma_{p_i}^k \in \{0, c_{p_i}\}, \; i = 1, n$$

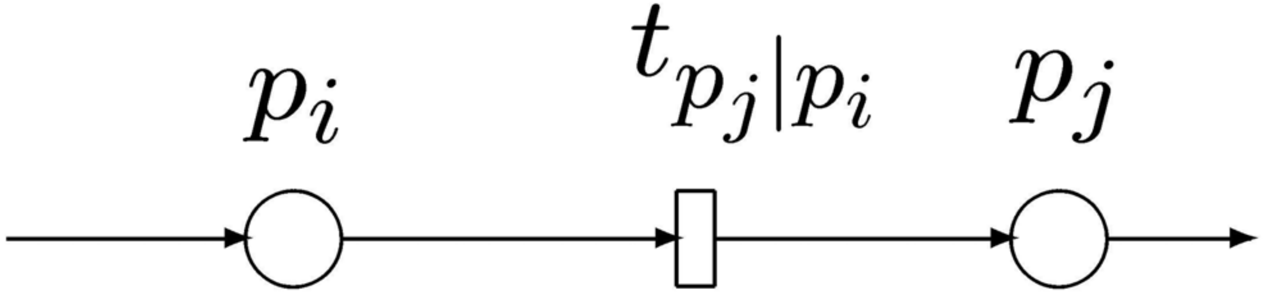
$$\mathbf{u}_k = (\gamma_{t_1}^k, \ldots, \gamma_{t_m}^k)^T$$

$$\gamma_{t_j}^k \in \{0, 1\}, \; j = 1, m$$

Reachability tree of the above introduced Petri net



Directed graphs (DG) in DEDS modelling



$$\gamma_{t_{\pi_i|\pi_j}}^{(k)} \in \{0,\ 1\}$$

$$\mathbf{X}(k+1) = \mathbf{\Delta}_k.\mathbf{X}(k) \quad , \quad k = 0, N$$

$$\mathbf{X}(k) = (\sigma_{\pi_1}^{(k)}(\gamma), ..., \sigma_{\pi_{n_{RT}}}^{(k)}(\gamma))^T, \; k = 0, N$$

$$\mathbf{\Delta}_k = \{\delta_{ij}^{(k)}\}_{n_{RT} \times n_{RT}}$$

$$\delta_{ij}^{(k)} = \gamma_{t_{\pi_i | \pi_j}}^{(k)}, \; i = 1, n_{RT}, \; j = 1, n_{RT}$$

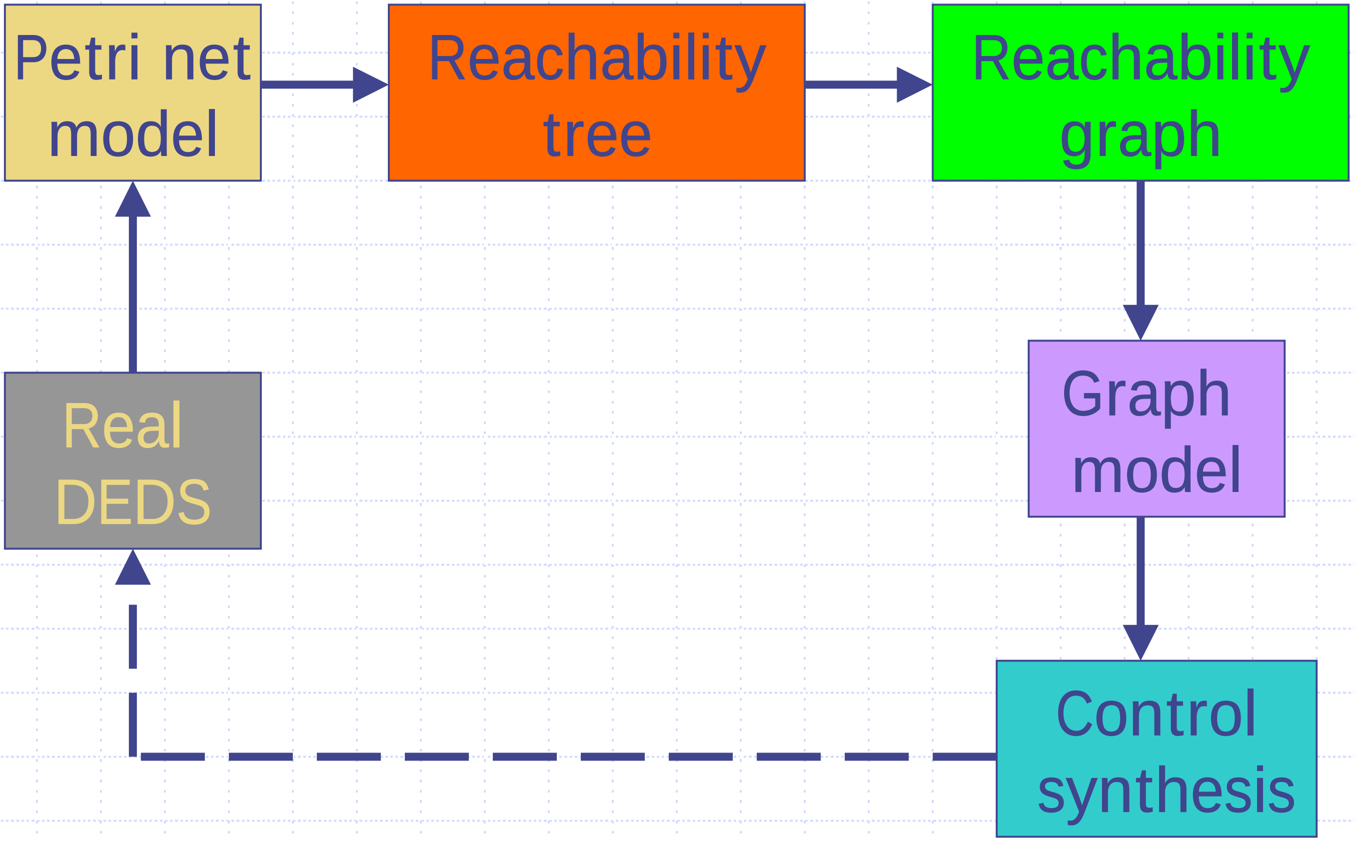
State machines

Petri nets where each transition has only one input and only one output position are named state machines.
They can be modelled by directed graphs (DG) without any problem.

Petri nets with general structure

In case of the general structure, when any transition is allowed to have more input positions and more output ones, the PN reachability graph has to be used.

Transforming the PN model to the DG model



DEDS control synthesis

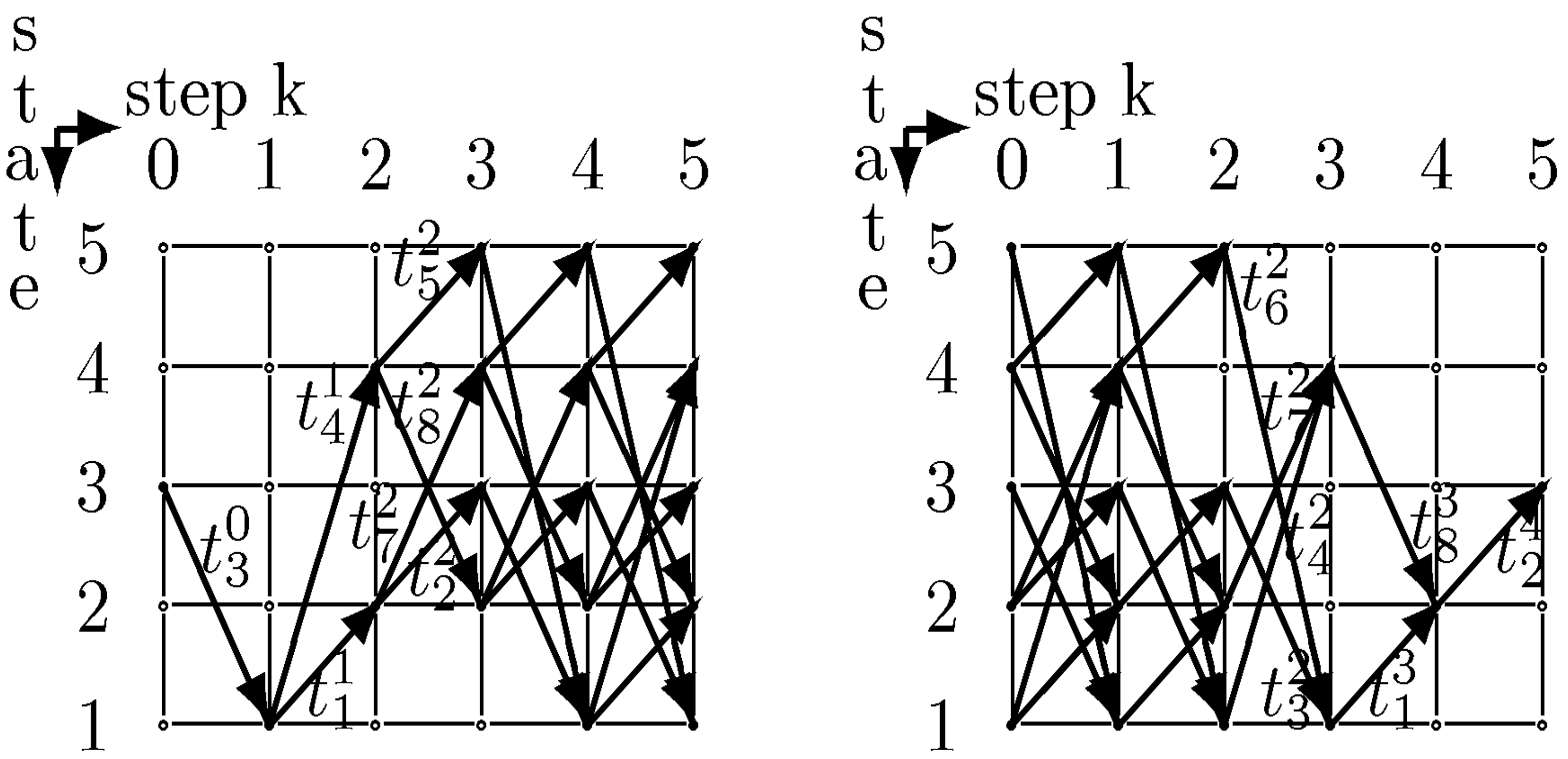
Definition of the control synthesis

Control synthesis = finding the most suitable sequence of discrete events (control interferences) which is able to ensure the transition (transformation) of the system from a given initial state into a prescribed terminal state at simultaneous fulfilling control task specifications that are imposed on the control task.

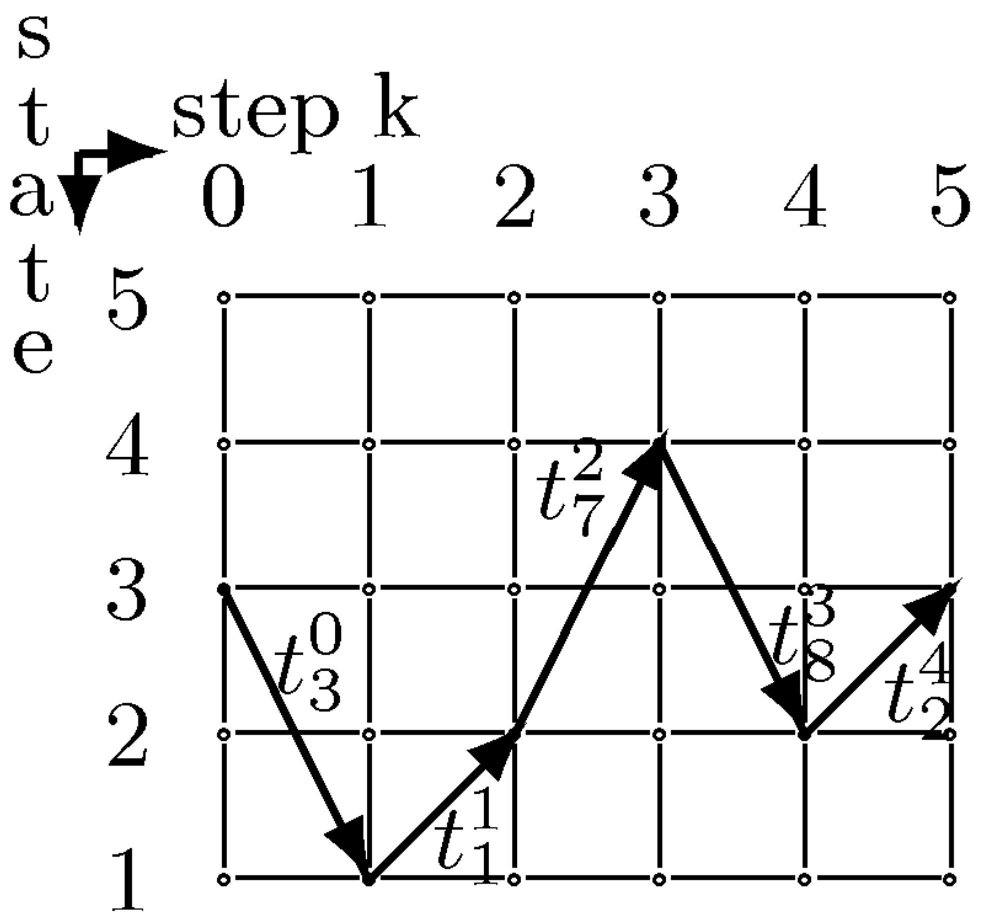
Control task specifications = criteria, constraints, etc. Usually, they are not given in analytical terms. Even, often they are given only verbally.

Basic principle of the proposed control synthesis method

Straight-lined reachability tree and the backtracking one



Intersection of the trees = state trajectory(-ies)



Procedure in analytical terms

- The staight-lined reachability tree (SLRT)

$$
\begin{aligned}
\{\mathbf{X}_1\} &= \mathbf{\Delta}.\mathbf{X}_0 \\
\{\mathbf{X}_2\} &= \mathbf{\Delta}.\{\mathbf{X}_1\} = \mathbf{\Delta}.(\mathbf{\Delta}.\mathbf{X}_0) = \mathbf{\Delta}^2.\mathbf{X}_0 \\
& \ldots \qquad \ldots \qquad \ldots \\
\{\mathbf{X}_N\} &= \mathbf{\Delta}.\{\mathbf{X}_{N-1}\} = \mathbf{\Delta}^N.\mathbf{X}_0
\end{aligned}
$$

- The backtracking reachability tree (BTRT)

$$
\begin{aligned}
\{\mathbf{X}_{N-1}\} &= \mathbf{\Delta}^T.\mathbf{X}_N \\
\{\mathbf{X}_{N-2}\} &= \mathbf{\Delta}^T.\{\mathbf{X}_{N-1}\} = (\mathbf{\Delta}^T)^2.\mathbf{X}_N \\
\ldots & \quad \ldots \quad \ldots \\
\{\mathbf{X}_0\} &= \mathbf{\Delta}^T.\{\mathbf{X}_1\} = (\mathbf{\Delta}^T)^N.\mathbf{X}_N
\end{aligned}
$$

The intersection of the SLRT and BTRT

$$
\begin{aligned}
\mathbf{M}_1 &= (\mathbf{X}_0, {}^1\{\mathbf{X}_1\}, \ldots, {}^1\{\mathbf{X}_{N-1}\}, {}^1\{\mathbf{X}_N\}) \\
\mathbf{M}_2 &= ({}^2\{\mathbf{X}_0\}, {}^2\{\mathbf{X}_1\}, \ldots, {}^2\{\mathbf{X}_{N-1}\}, \mathbf{X}_N) \\
\mathbf{M} &= \mathbf{M}_1 \cap \mathbf{M}_2 \\
\mathbf{M} &= (\mathbf{X}_0, \{\mathbf{X}_1\}, \ldots, \{\mathbf{X}_{N-1}\}, \mathbf{X}_N)
\end{aligned}
$$

Using the principle of causality

Due to the principle of causality any shorter feasible solution is involved in the longer feasible one. Hence, when

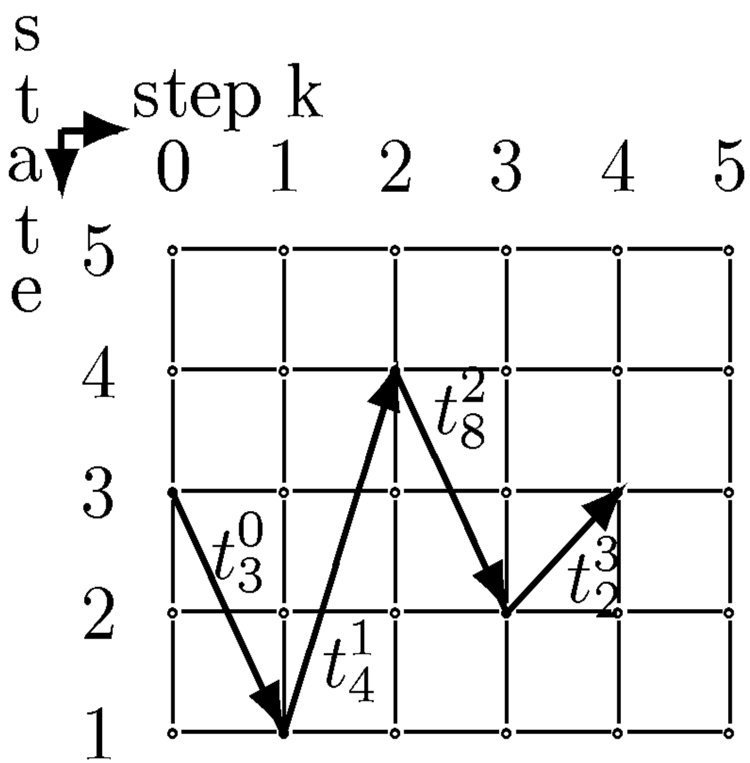
$\mathbf{M}_2$ is shifted to the left before the intersection.

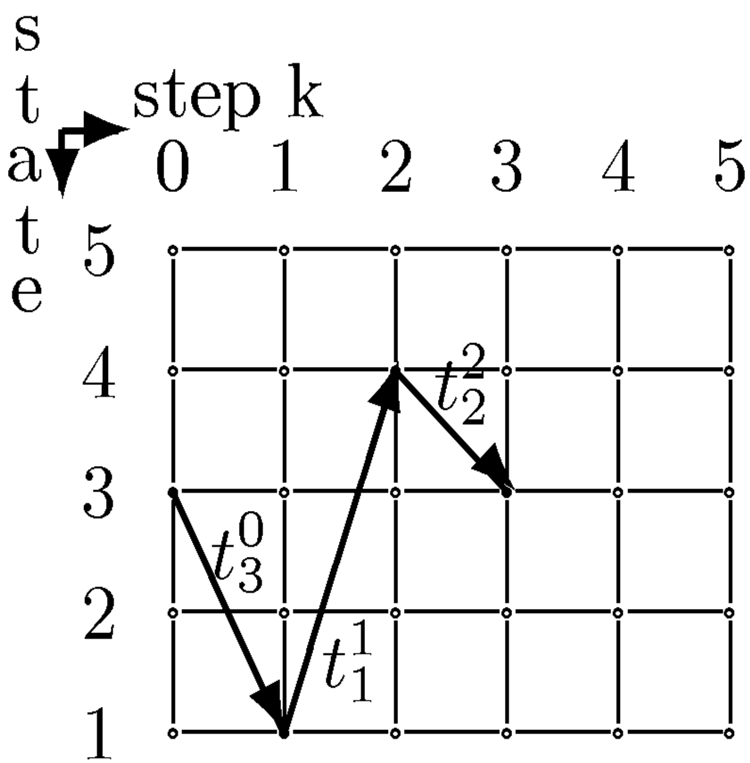
$$^{-1}\mathbf{M} = (\mathbf{x}_0, \{\mathbf{x}_1\}, \ldots, \{\mathbf{x}_{N-2}\}, \mathbf{x}_{N-1}) \qquad (18)$$

where $\mathbf{x}_{N-1} = \mathbf{x}_t$.

Shifting (finding the $(n \times (N-k+1))$ matrices $^{-k}\mathbf{M}, k = 1, 2, ...$) can continue until the intersections exists, i.e. until $\mathbf{x}_0 \in {}^2\{\mathbf{x}_k\}$ and $\mathbf{x}_t \in {}^1\{\mathbf{x}_{N-k}\}$.

Shorter trajectories obtained by shifting





MATLAB procedure for enumerating the RT

```
Xreach=x0
Art=[0]
[n,m]=size(F);
B=Gt-F
i=0
while i < size(Xreach,2)
        i=i+1;
        for k=1:m
            x(k)=all(Xreach(:,i) >= F(:,k));
        end
        findx=find(x)
        for k=1:size(findx,2)
```

```
            bb = Xreach(:,i)+B(:,findx(k));
            matrix=[];
            for j=1:size(Xreach,2)
                matrix=[matrix,bb];
            end;
            v=all(matrix == Xreach);
            j=find(v);
            if any(v)
                Art(i,j)= findx(k);
            else
                Xreach=[Xreach,bb];
                Art(size(Art,1)+1,size(Art,
                Art(i,size(Art,2))=findx(k)
            end;
        end;
Xreach;
Art;
end
```

Example 1 – Two agents cooperation

The agent A needs to do the activity (i.e. to solve a problem) P. However, A is not able to do P. Consequently, A requests the agent B to do P for him.

The places of the PN-based model:

p1 – A wants to do P
p2 - A waits for an answer from B
p3 - A waits for a help from B
p4 - the failure of the cooperation
p5 - the satisfying cooperation
p6 - A requests B to do P
p7 - B refuses to do P

p8 - B accepts the request of A to do P

p9 - B is not able to do P

p10- doing P by B

P11- B receives the request of A

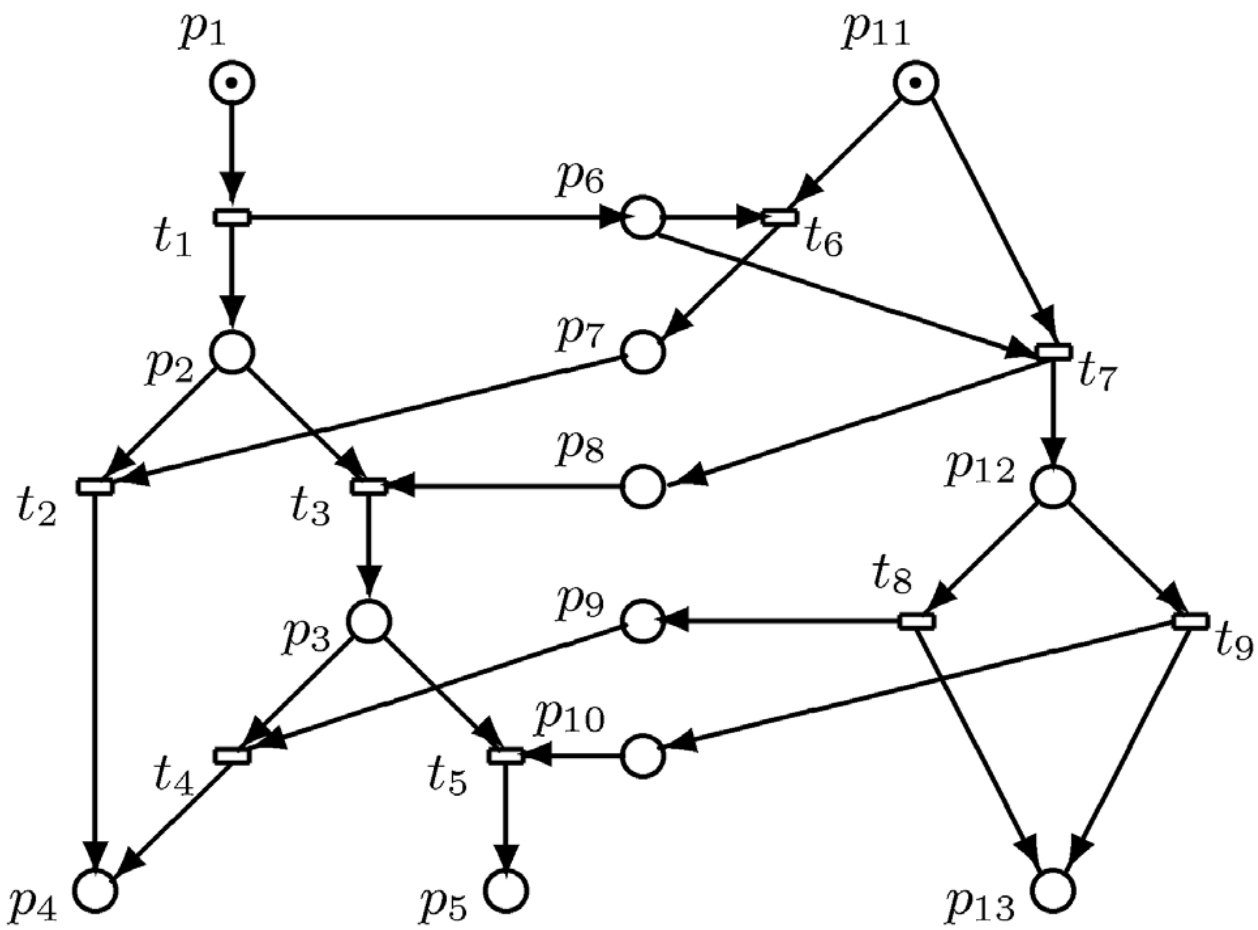
p12- B is willing to do P for A

p13- the end of the work of B

The transitions of the PN-based model:

t1 – t9 represent discrete events realizing the system dynamics

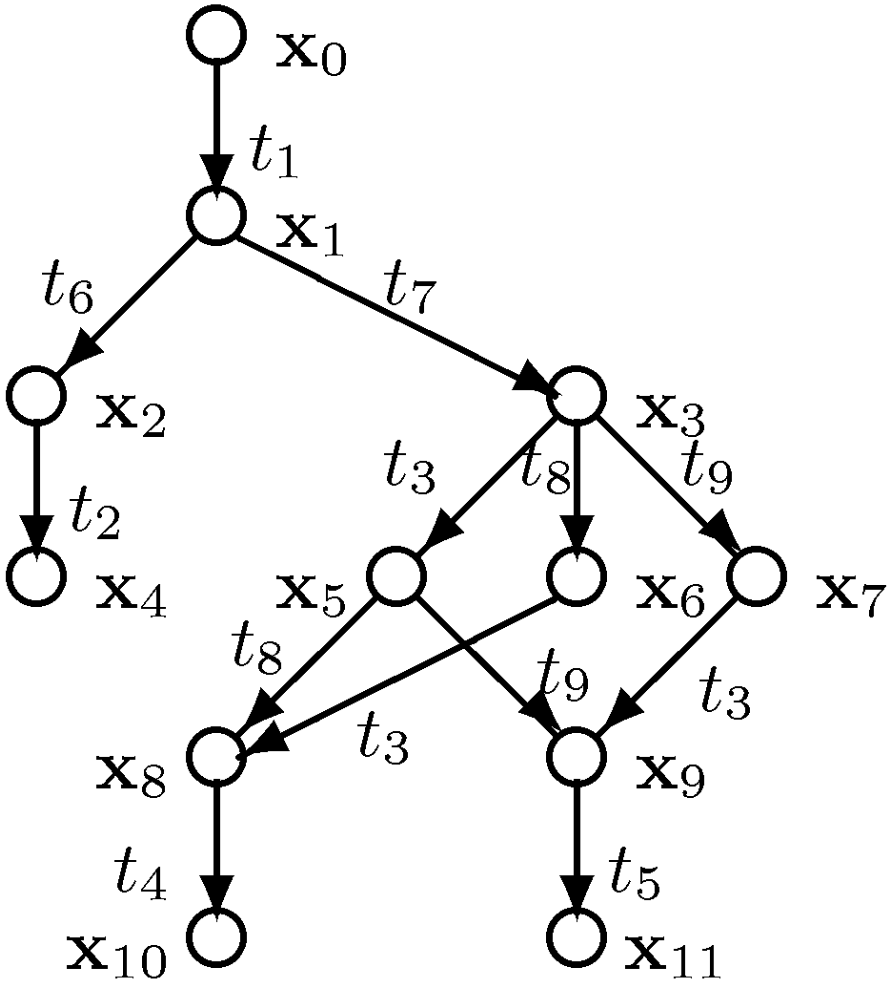
PN-based model



Enumerated RT

$$\mathbf{A}_k = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 8 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The graphical expression of the RT



The space of reachable states

$$\mathbf{X}_{reach} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Control synthesis

The initial state

$$\mathbf{x}_0 = (1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0)^T$$

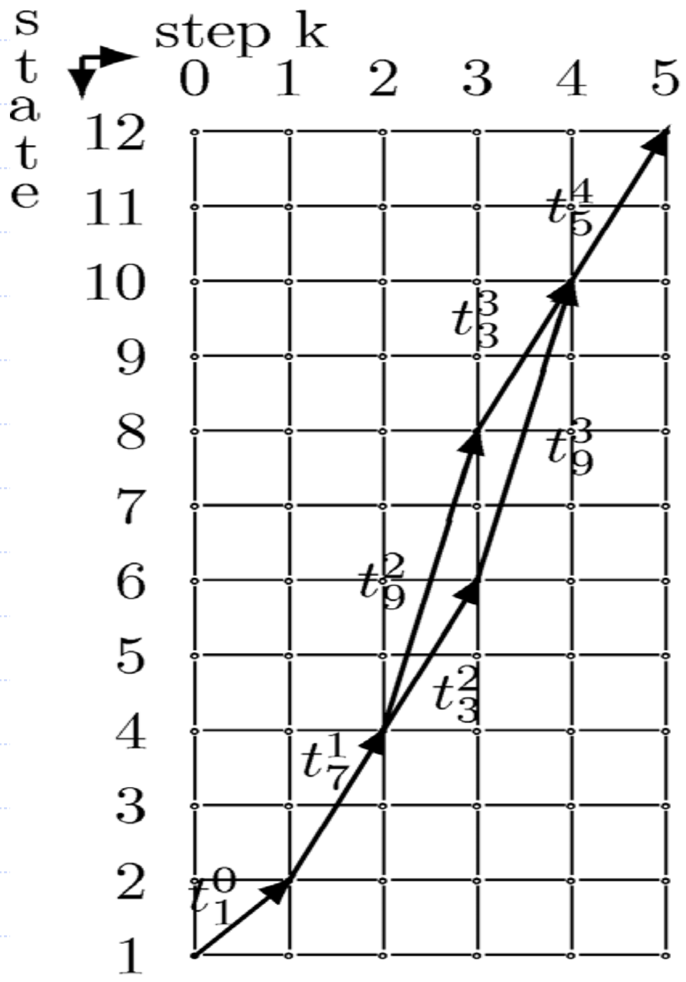
The terminal state – the successful cooperation

$$\mathbf{x}_N = (0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1)^T$$

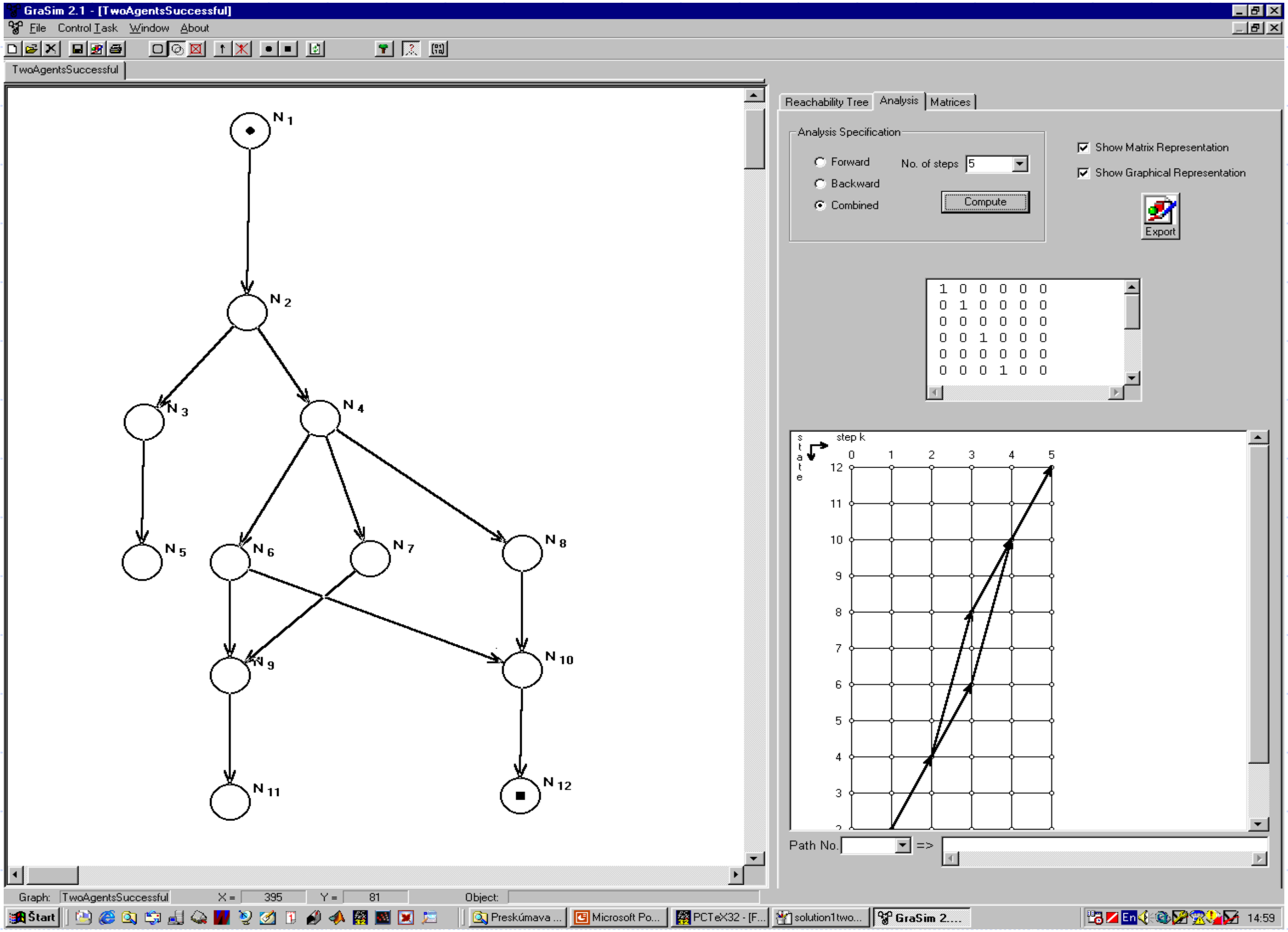
The intersection of the SLRT and BTRT

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

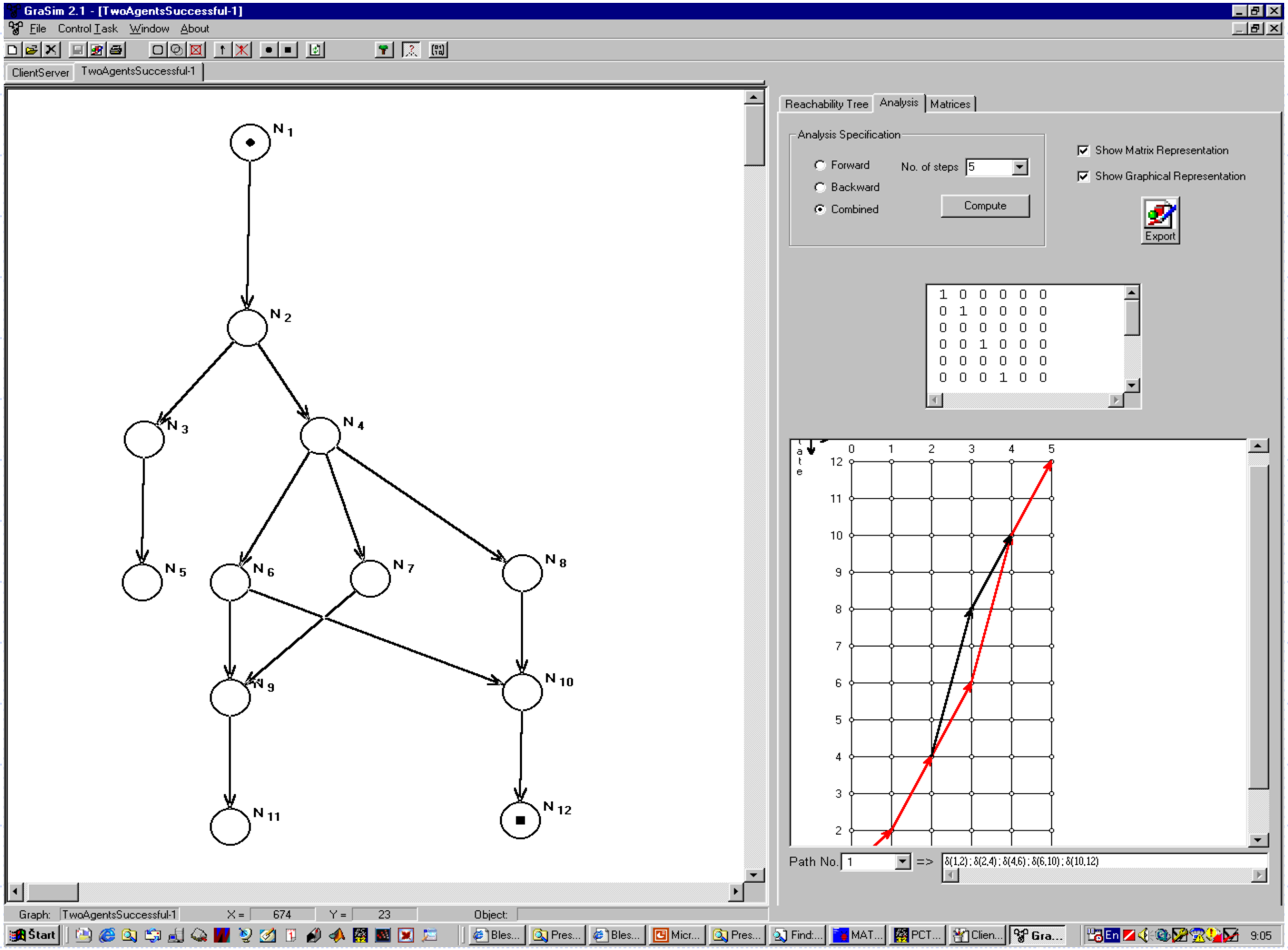
The state trajectories – the successful cooperation



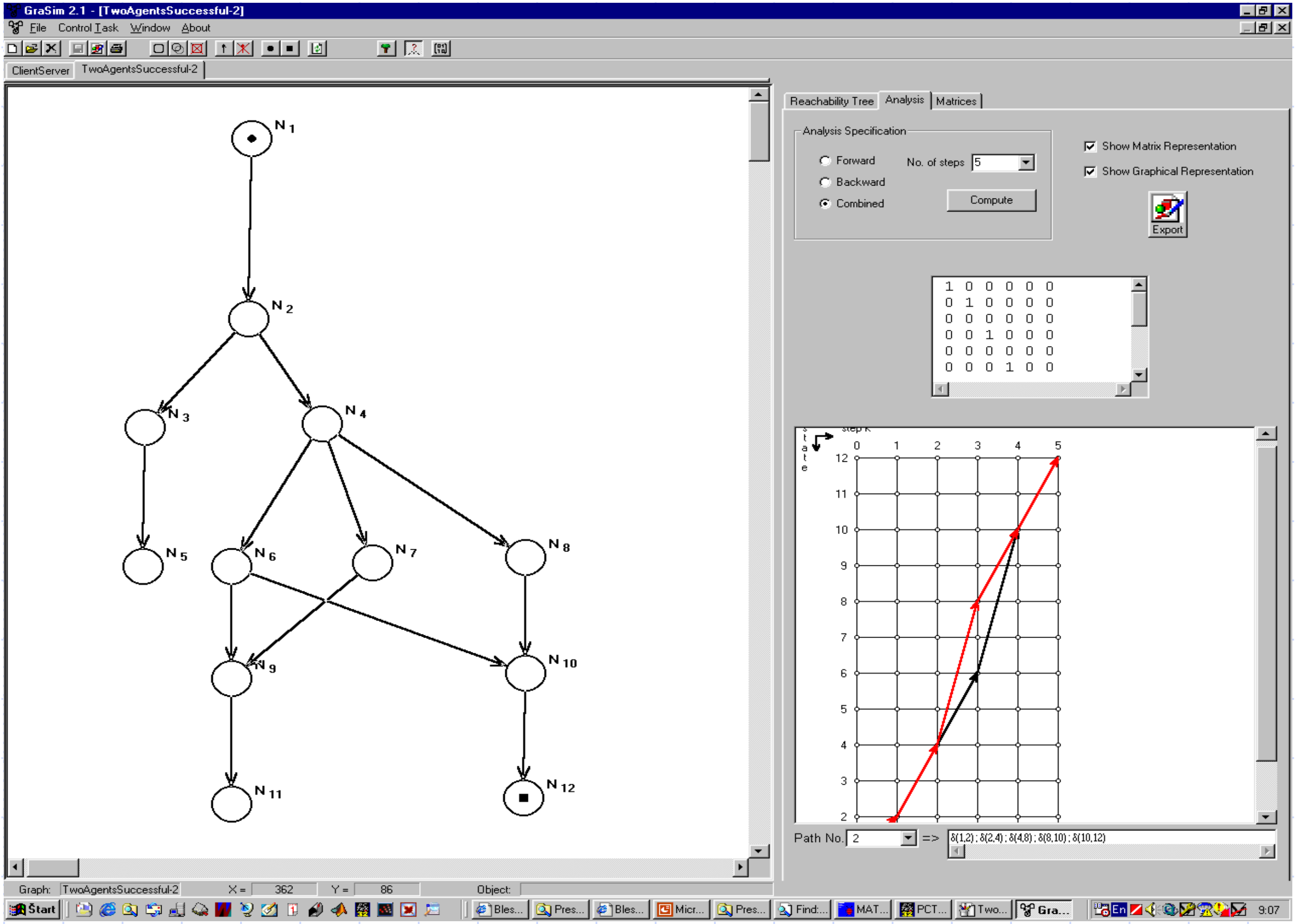
Graphic tool - GraSim



Succesfull cooperation 1

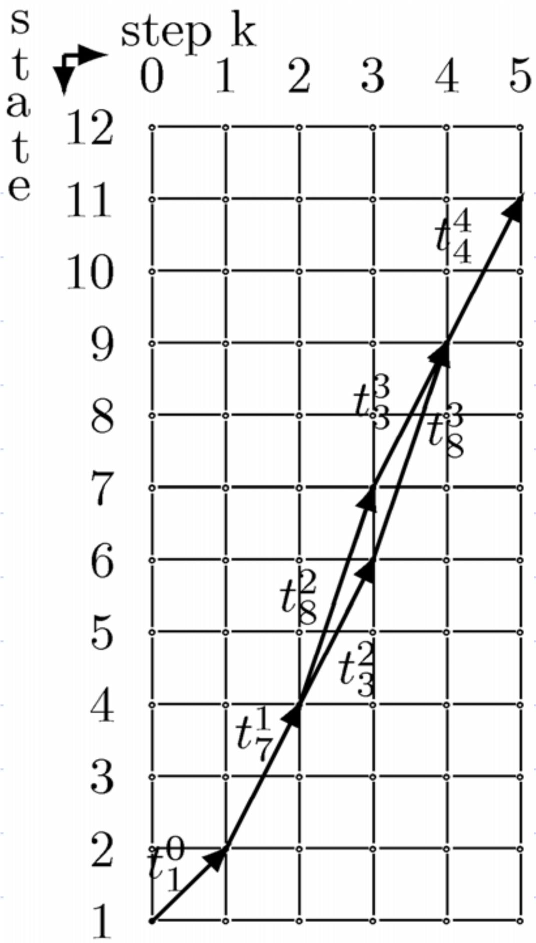


Succesfull cooperation 2



The failured cooperation – when B is not able to do P

$$\mathbf{x}_N = (0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1)^T$$



PN models with general structure and dynamics

In general, there are two kinds of the PN models with the general structure and dynamics:

- The PN models with the finite space of reachable states (like the previous example of two agents cooperation)
- The PN models with the infinite space of reachable states (like the next example of FMS)

Example 2 – The flexible manufacturing system

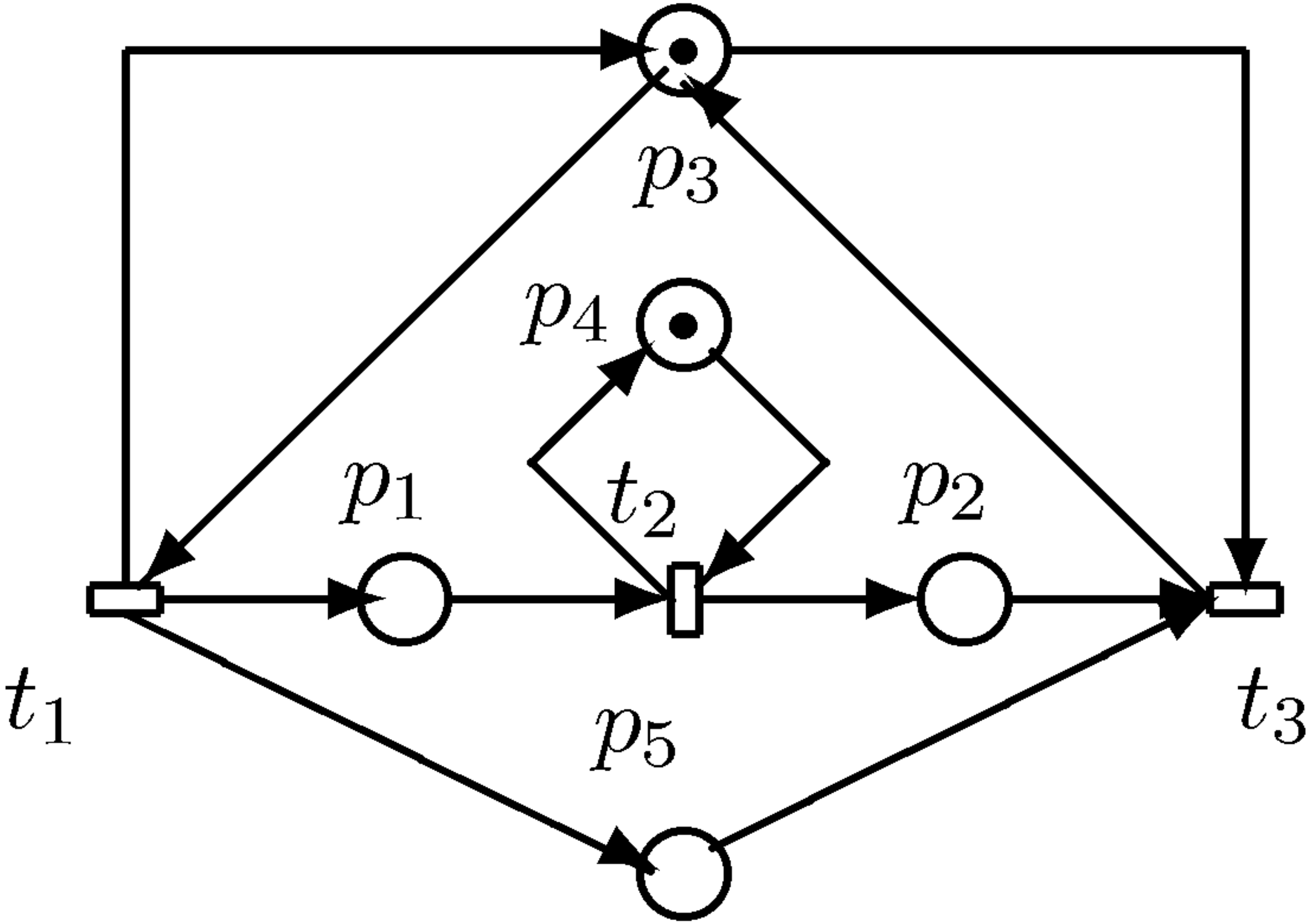
Consider the robotic cell with two conveyors C1, C2, the NC-machine M, with buffer B (having the input part B1 and the output part B2), and the robot R.

Defining the PN places and transitions:

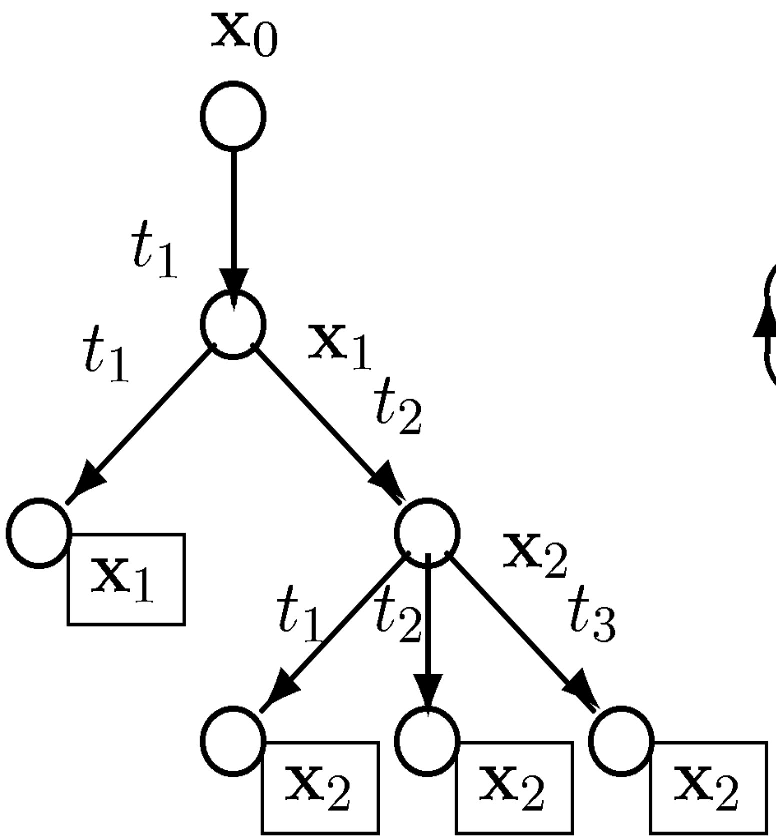
p1 = waiting the input parts
p2 = waiting the output parts
p3 = R is available
p4 = M is available
p5 = contents of B

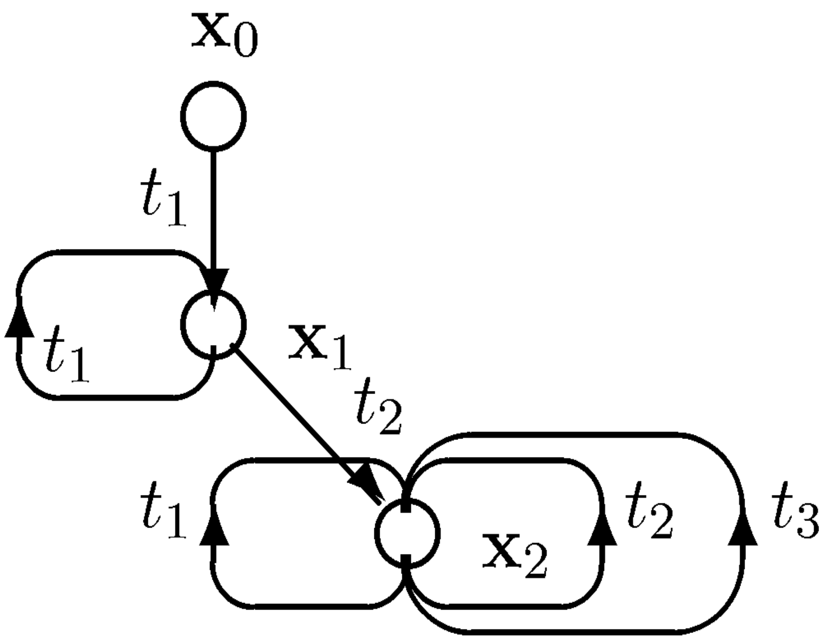
t1 = taking from C1 by R
t2 = machining by M
t3 = putting on C2 by R

The PN-based model of the FMS



The reachability tree and reachability graph





The model parameters

$$\mathbf{F} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{x}_0 = (0,\ 0,\ 1,\ 1,\ 0)^T$$

The RT and state space

$$\mathbf{A}_k = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & (1,2,3) \end{pmatrix} \quad \boldsymbol{\Delta}_k = \begin{pmatrix} 0 & 0 & 0 \\ t_1 & t_1 & 0 \\ 0 & t_2 & (t_1, t_2, t_3) \end{pmatrix}$$

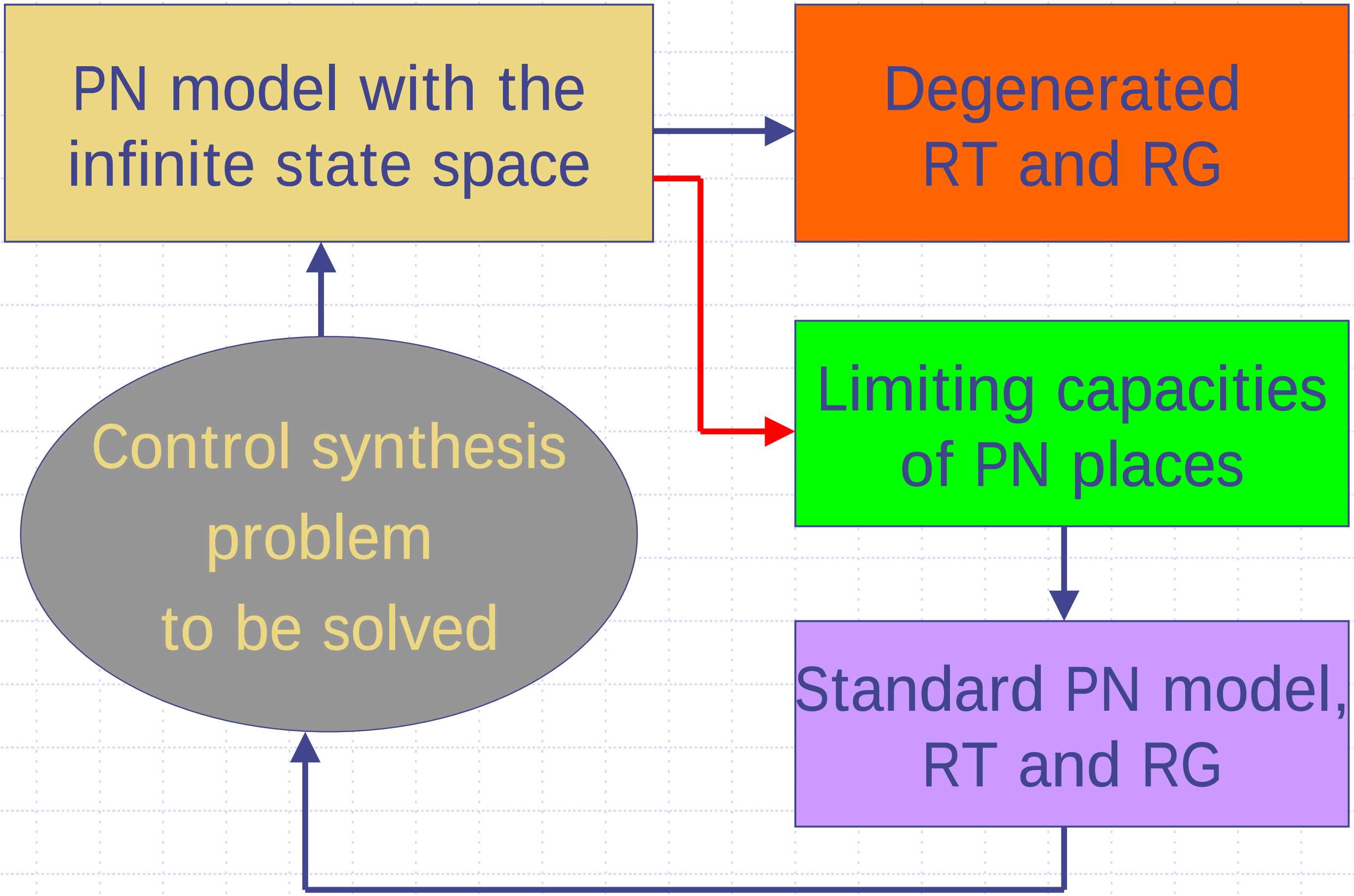
$$\mathbf{X}_{reach} = \begin{pmatrix} 0 & \omega & \omega \\ 0 & 0 & \omega \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & \omega & \omega \end{pmatrix}$$

The ambiguity and how to deal with it

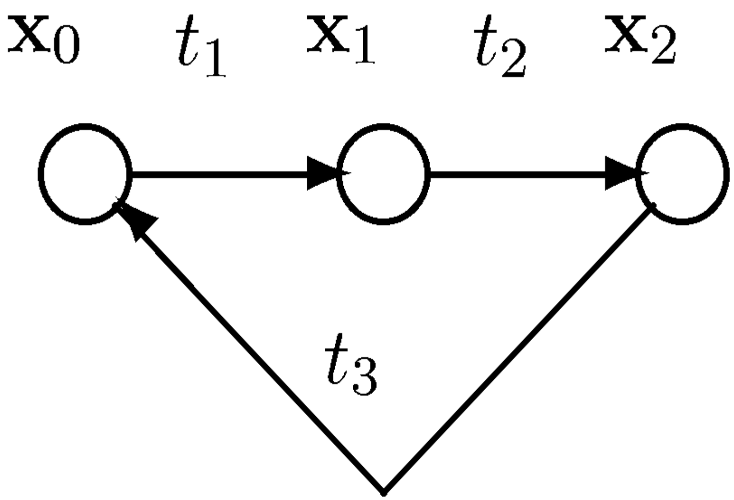
When the capacities of the PN places are infinite, there is the ambiguity as to the elements of the matrix A. The cycles engender in the RT and RG. The state space of the reachable states is infinite. Infinity is expressed by the symbol ω

Hence, in order to find a reasonable solution, the finite capacities of the PN places have to be determined.

Dealing with the ambiguity



The finite capacity $c_{p_5} = 1$

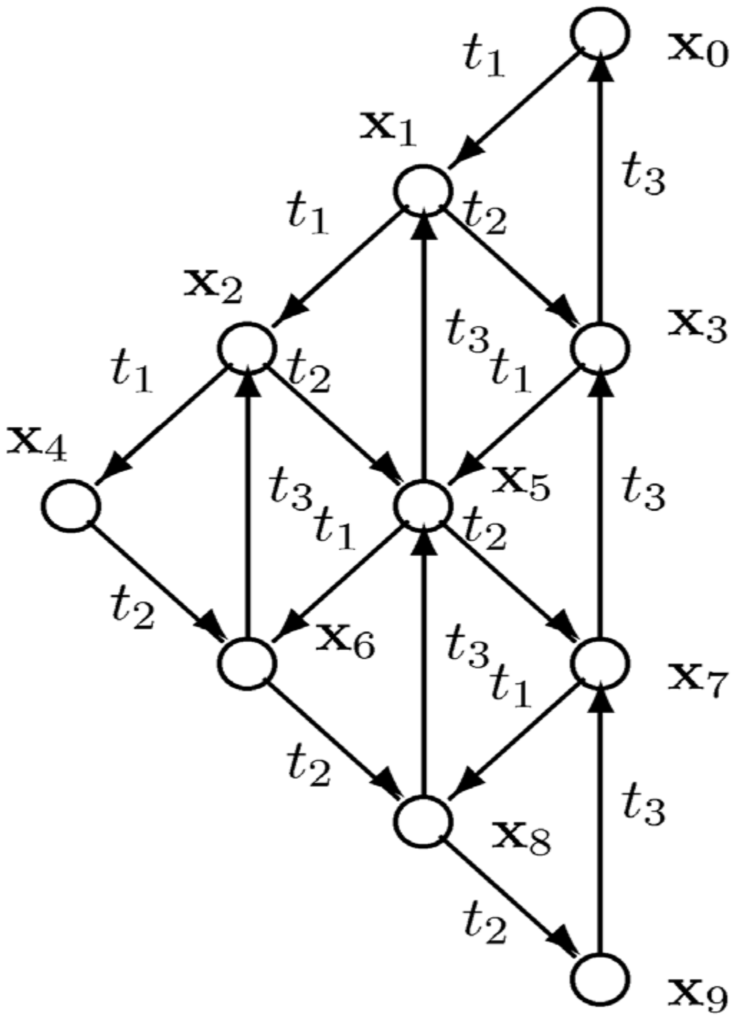


$$\mathbf{A}_k = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 3 & 0 & 0 \end{pmatrix} \quad \mathbf{X}_{reach} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

The finite capacities $c_{p_i} = 3,\ i = 1,\ 2,\ 5$

$$\mathbf{A}_k = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \end{pmatrix}$$

The reachability graph



The state space of reachable states

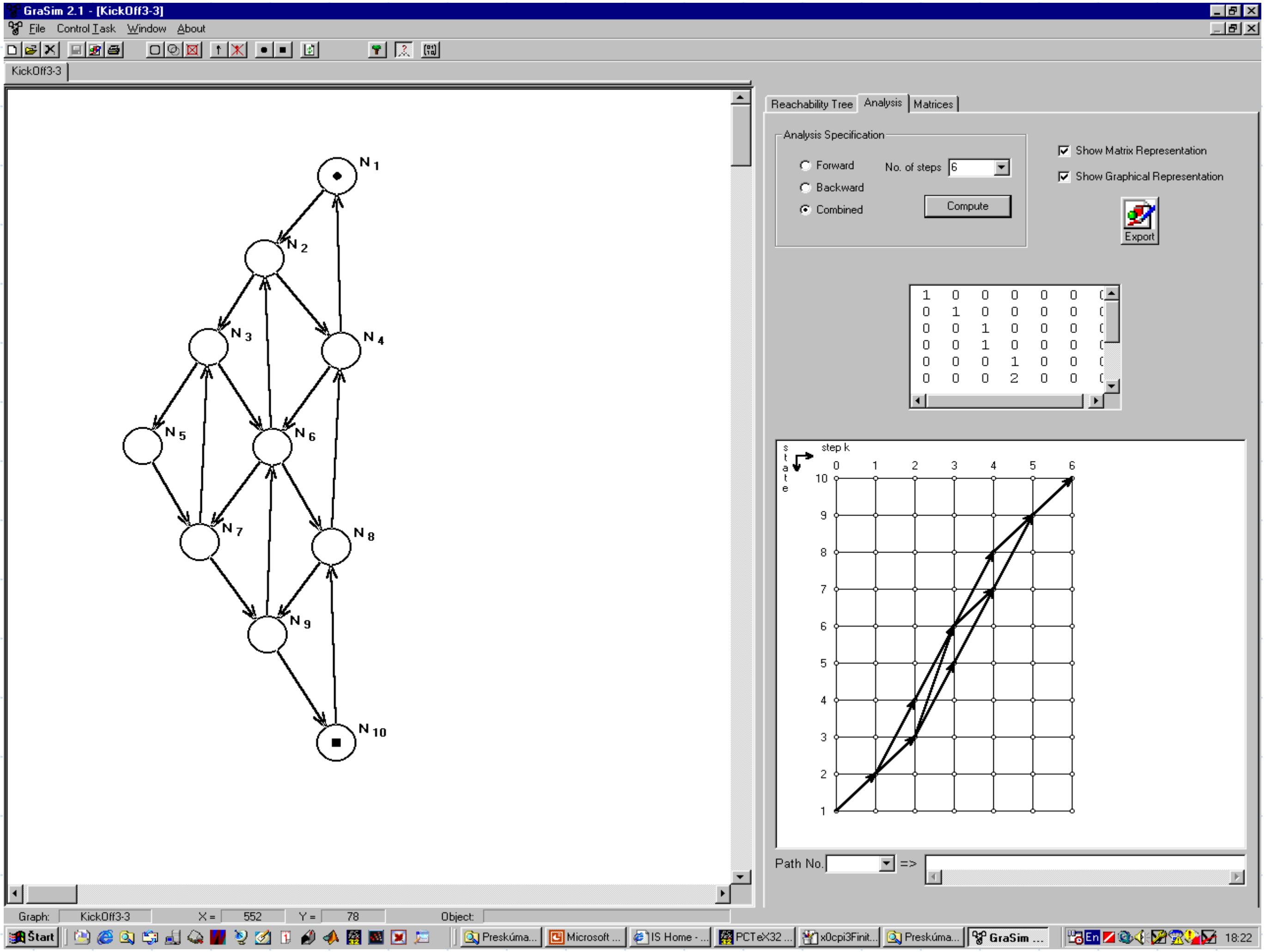
$$\mathbf{X}_{reach} = \begin{pmatrix} 0 & 1 & 2 & 0 & 3 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 2 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 & 3 & 2 & 3 & 2 & 3 & 3 \end{pmatrix}$$

The control synthesis

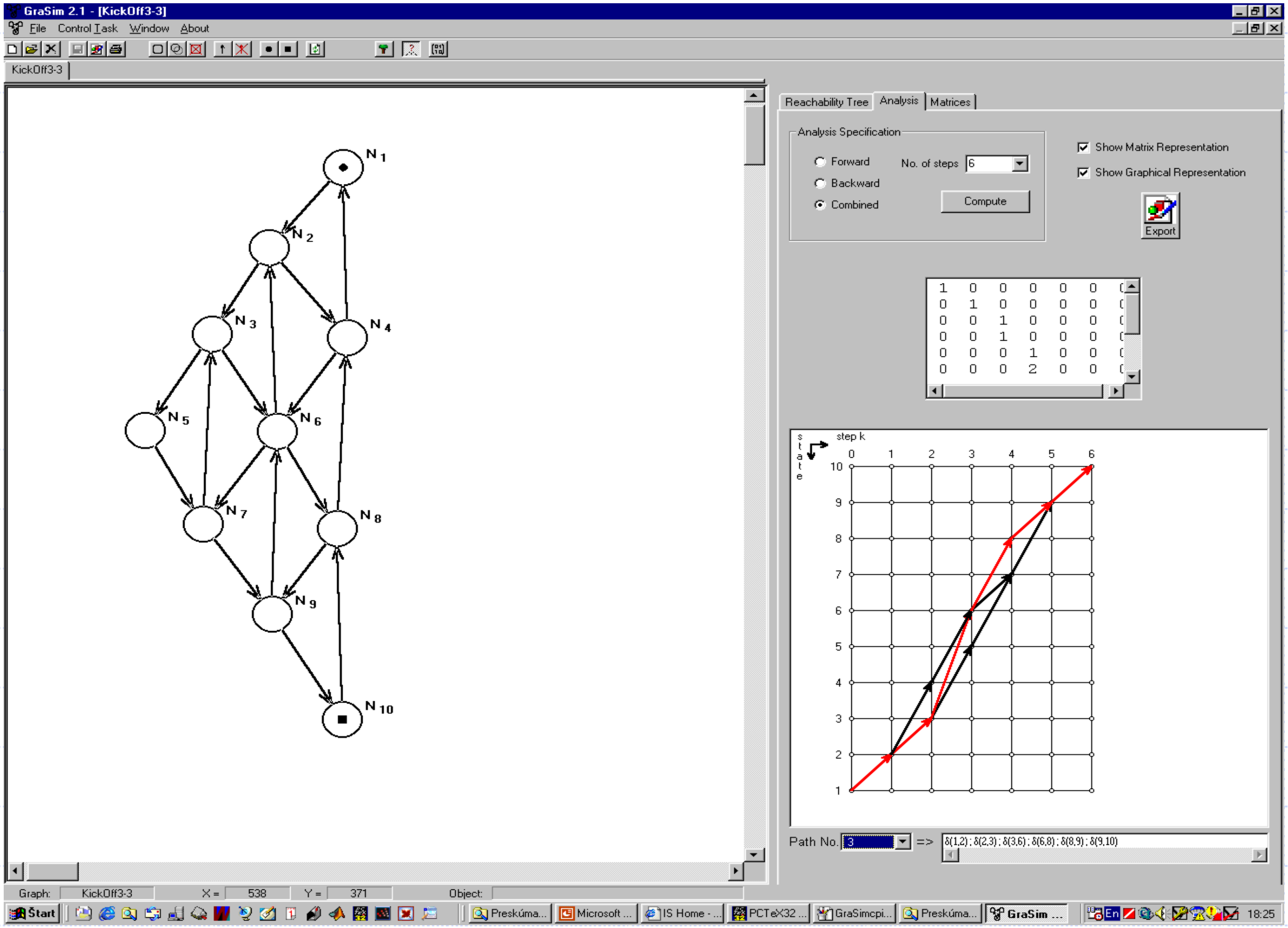
$$\mathbf{x}_0 = (0,\ 0,\ 1,\ 1,\ 0)^T \qquad \mathbf{x}_9 = (0,\ 3,\ 1,\ 1,\ 3)^T$$

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

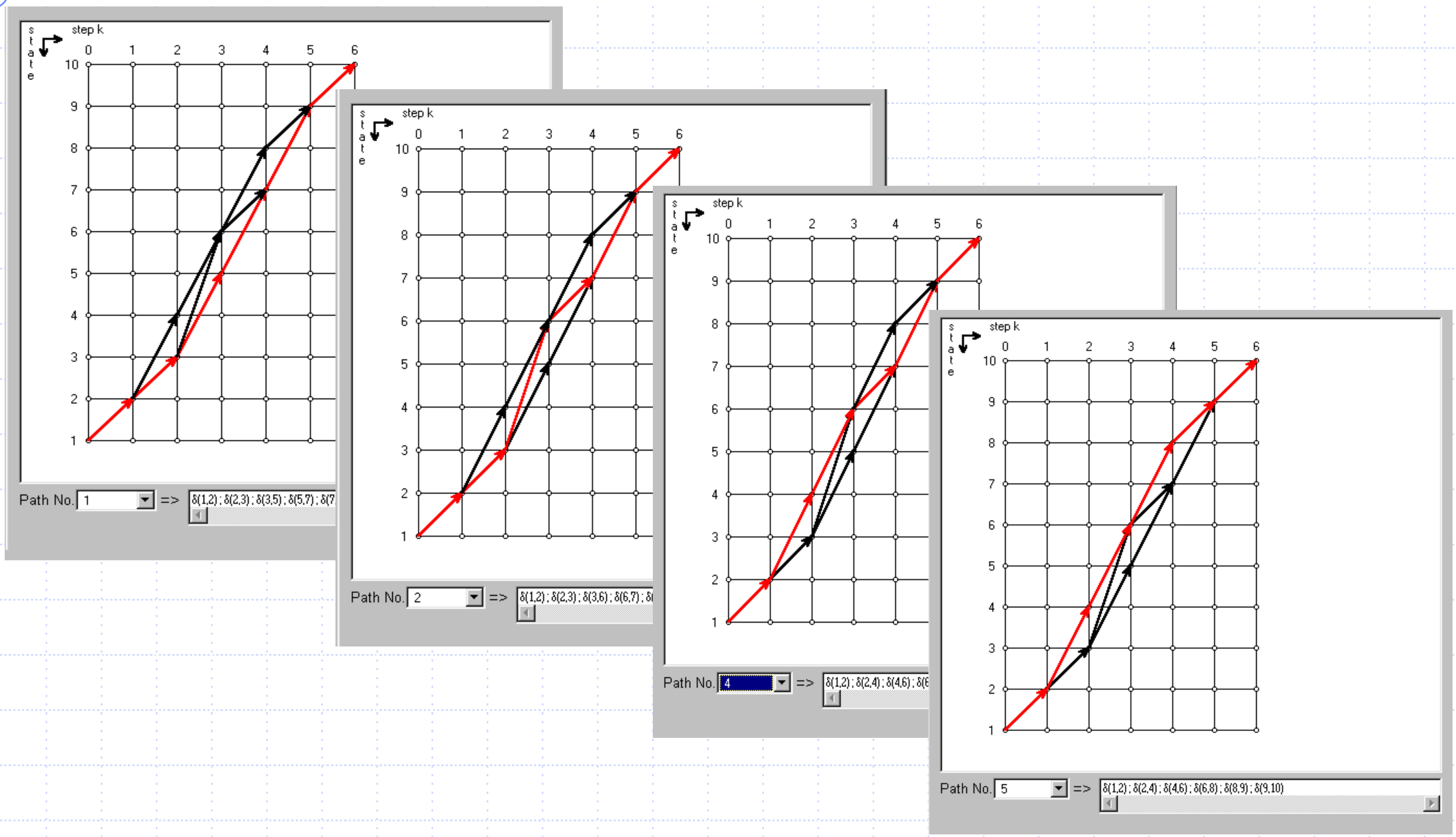
Using the system GraSim



The trajectory No. 3



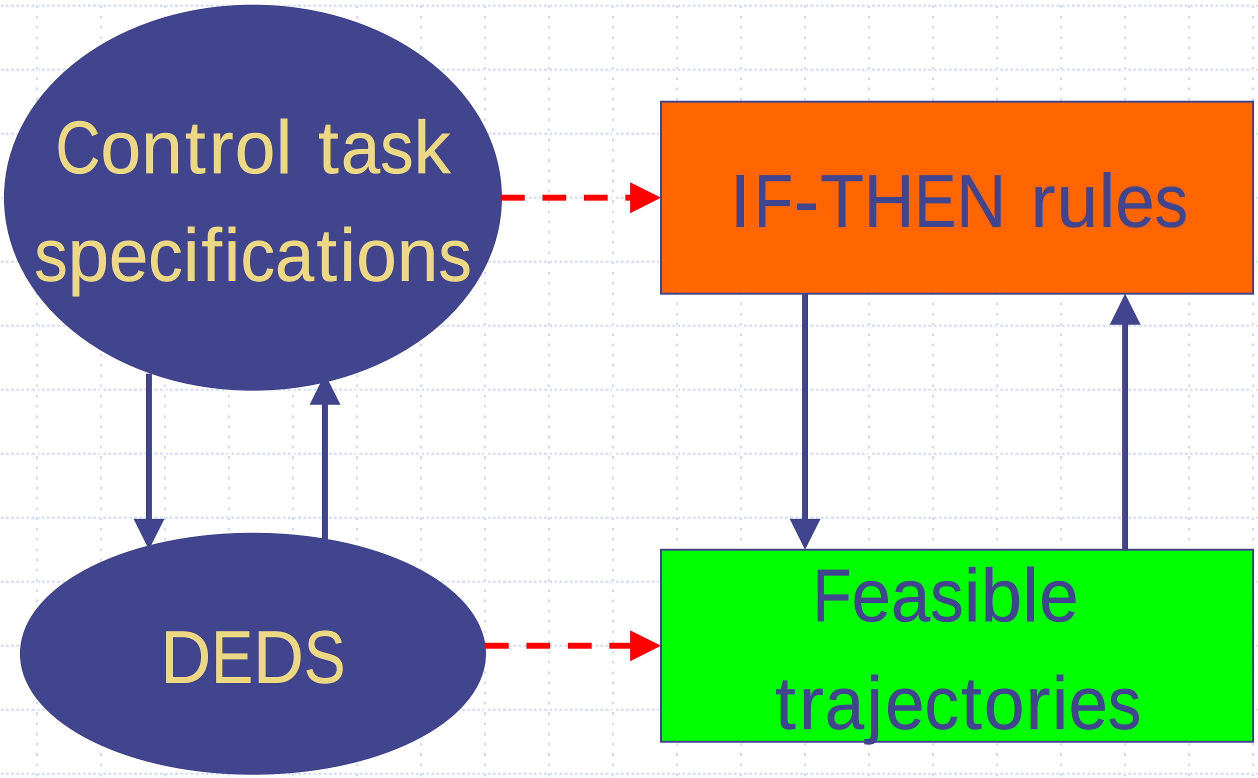
The other trajectories



Intelligent Control Synthesis

DEDS control task specifications are usually given in non-analytical terms, often only verbally. Knowledge-based approaches have to be used in order to choose the most suitable trajectory. The knowledge base (KB) expressing the control task specifications in the form of IF-THEN rules can be modelled by means of the logical and/or fuzzy PN. Thus, the KB can be expressed in analytical terms analogically to the PN-based model of DEDS. The inference mechanism can be described in analytical terms as well. The author’s approach how to do was presented recently.

Knowledge-based choice of the trajectory



Conclusions

- Simple general method of DEDS modelling and control synthesis was presented
- Its applicability to DEDS with general structure and dynamics was demonstrated
- Two different kinds of DEDS were investigated as to the control synthesis:
  - DEDS having the PN model with finite state space (like the case of the two agents cooperation)
  - DEDS having the PN model with infinite state space (like the flexible manufacturing system)

- It was pointed out how to deal with the control synthesis of DEDS having the PN model with infinite state space.

Future work on this way

- To innovate the method permanently to extend its reasonable applicability for larger and larger class of DEDS able to be modelled by Petri nets
- To find new simulation procedures and tools