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Petri net-based modelling and simulation of agent systems

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Contents

1. Introduction

- The agent behaviour is understood here to be discrete event dynamic system
- Discrete event dynamic systems (DEDS) are frequently modelled by means of Petri nets
- Petri nets (PN) yield exact mathematical model of the DEDS in question
- Place/transition PN (P/T PN) offer the model of DEDS in the form of the linear discrete system

- The reachability tree (RT) of a P/T PN represents the space of reachable states. These states (*the RT leaves*) are reachable from a given initial state (*the RT root*).
- Define the straight-lined reachability tree (SLRT) and the backtracking reachability tree (BTRT). The root of SLRT is a DEDS initial state $\mathbf{x}_0$ and the root of BTRT is a DEDS terminal state $\mathbf{x}_N$
- The intersection of both the SLRT and the BTRT yields the set of feasible trajectories from the initial state $\mathbf{x}_0$ to the terminal state $\mathbf{x}_N$

P/T PN structure

$\langle P, T, F, G \rangle$ ... bipartite graph

$P = \{p_1, p_2, \ldots, p_n\}$ ... places

$T = \{t_1, t_2, \ldots, t_m\}$ ... transitions

$F \subseteq P \times T \ ; \ G \subseteq T \times P$

$P \cap T = \emptyset \ ; \ F \cap G = \emptyset$

P/T PN dynamics

$X_1 = \mathbf{x}_0$

$\langle X, U, \delta, \mathbf{x}_0 \rangle$ … dynamics

$X = \{\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_N\}$ …state vectors

$U = \{\mathrm{U}_1, \mathrm{U}_2, \ldots, \mathrm{U}_M\}$…control vectors

$\delta : X \times U \rightarrow X$ … transition function

$\mathbf{x}_0$ … initial state vector ($\mathrm{X}_1 = \mathbf{x}_0$)

2. PN-based modelling DEDS

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{B}.\mathbf{u}_k, \quad \mathrm{k} = 0, 1, \ldots$$

$$\mathbf{B} = \mathbf{G}^T - \mathbf{F}$$

$$\mathbf{F}.\mathbf{u}_k \leq \mathbf{x}_k, \quad k = 0, 1, \ldots$$

$$\mathbf{x}_k = (\sigma^k{}_{p_1}, \sigma^k{}_{p_2}, \ldots, \sigma^k{}_{p_n})^T$$

$$\sigma^k{}_{p_i} \in \{0, 1, \ldots, c_{p_i}\}$$

$$\mathbf{u}_k = (\gamma^k{}_{t_1}, \gamma^k{}_{t_2}, \ldots, \gamma^k{}_{t_m})^T$$

$$\gamma^k{}_{t_j} \in \{0,1\}$$

$\mathbf{F}$ … $(n \times m)$ incidence matrix

it corresponds to the set $F \subseteq P \times T$

$$f_{ij} \in \{0, 1, \ldots, M_{f_{ij}}\}, \; i = 1, 2, \ldots, n;$$

$$j = 1, 2, \ldots, m$$

$\mathbf{G}$ … $(m \times n)$ incidence matrix

it corresponds to the set $G \subseteq T \times P$

$$g_{ij} \in \{0, 1, \ldots, M_{g_{ij}}\}, \quad i = 1,2,\ldots,m;$$

$$j = 1,2,\ldots,n$$

3. The reachability tree & graph

$G_{rt} = (V_{rt}, E_{rt})$ … reachability tree

$V_{rt} = \{v_0, v_1, \ldots, v_{N_r}\}$ … RT nodes

$v_i, \ i = 0,1\ldots,N_r$ represent the state vectors $\mathbf{x}_i, i = 0,1,\ldots,N_r$

$E_{rt} = \{e_1, e_2, \ldots, e_M\}$ … RT edges

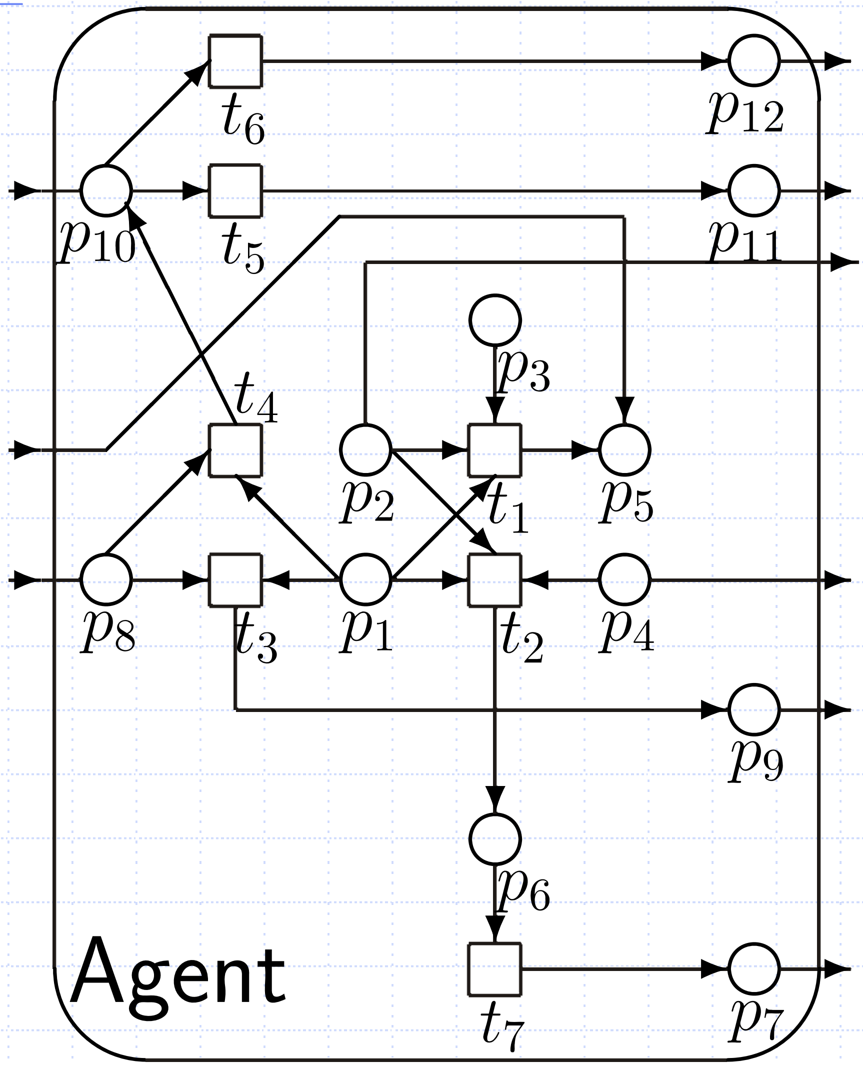
Two RT nodes $v_i, v_j \in V$ are connected by the oriented arc $e = e_{v_i \to v_j} \in E$ marked by the transition $t = t_{v_i \to v_j} = t_{\mathbf{x}_i \to \mathbf{x}_j} \in T$

For P/T PN represented by $\mathbf{F}, \mathbf{G}, \mathbf{x}_0$ the RT is represented by $\mathbf{A}_{rt}, \mathbf{X}_{reach}$

$\mathbf{A}_{rt}$ is the $(N \times N)$, $N = N_r + 1$, quasi-functional adjacency matrix

$\mathbf{X}_{reach}$ columns are $\mathbb{X}_i$, $i=1, 2, \ldots, N$

4. The PN-based model of an agent



$$P = \{p_1, p_2, \ldots, p_{12}\}$$

- p_1 - the agent A is free
- p_2 - a problem P_A has to be solved by A
- p_3 - A is able to solve P_A
- p_4 - A is not able to solve P_A
- p_5 - P_A is solved
- p_6 - P_A cannot be solved by A; another agent has to be be contacted
- p_7 - A asks another agent(s) for help to solve P_A

p_8 - *A* is asked by another agent(s) to solve a problem P_B

p_9 - *A* refuses the help

p_{10} - *A* accepts the request of another agent(s) for help

p_{11} - *A* is not able to solve P_B

p_{12} - *A* is able to solve P_B

The PN transitions $t_j \in T = \{t_1, t_2, ..., t_7\}$ represent the discrete events expressing the starting and/or ending the activities

PN-model parameters

$$\mathbf{F} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{G}^T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Initial conditions and RT

a) $\mathbf{x}_0 = (1,1,1,0,0,0,0,0,0,0,0,0)^T$ — A is able to solve P_A

b) $\mathbf{x}_0 = (1,1,0,1,0,0,0,0,0,0,0,0)^T$ — A is not able to solve P_A

c) $\mathbf{x}_0 = (1,0,0,0,0,0,0,1,0,0,0,0)^T$ — A is asked by another agent for help

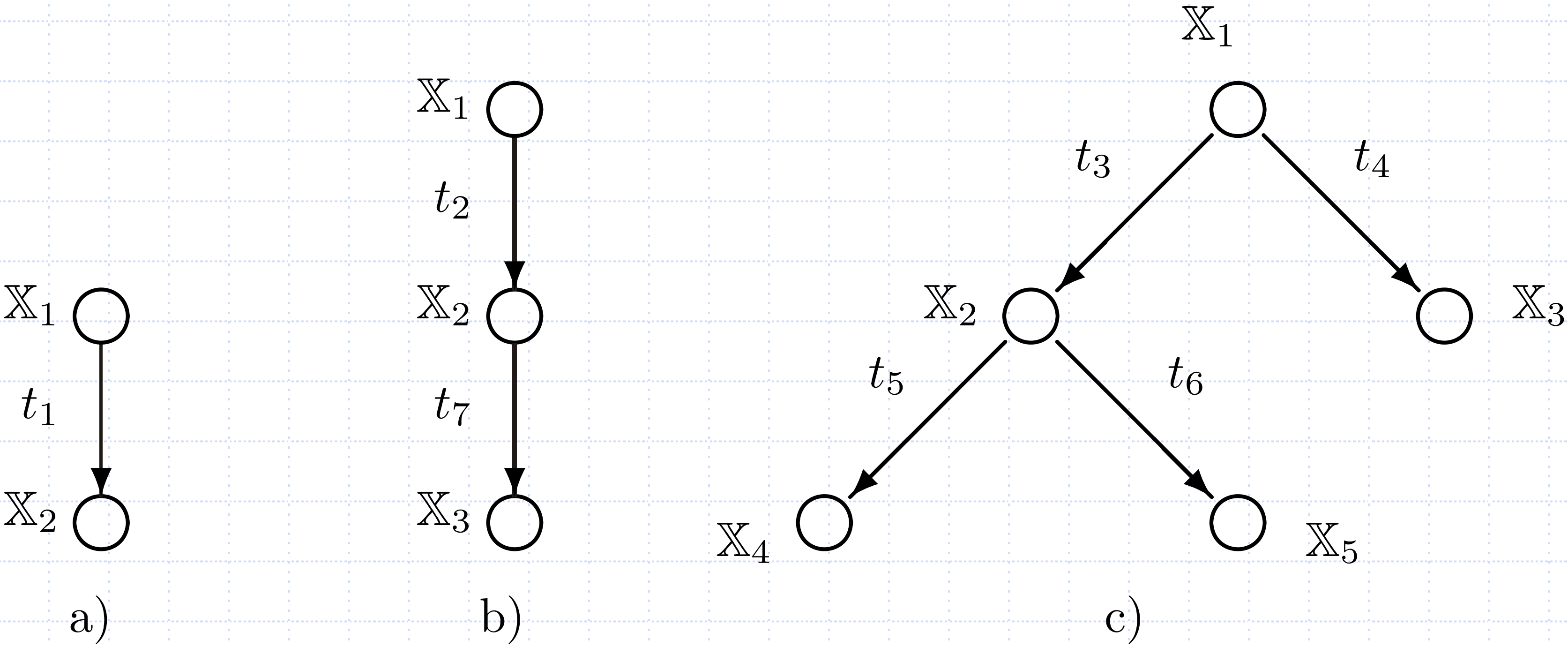
$$\mathbf{A}_{rt} = \begin{pmatrix} 0 & 3 & 4 & 0 & 0 \\ 0 & 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{X}_{reach} = (\mathbb{X}_1, \mathbb{X}_2, \ldots, \mathbb{X}_5)^T$$

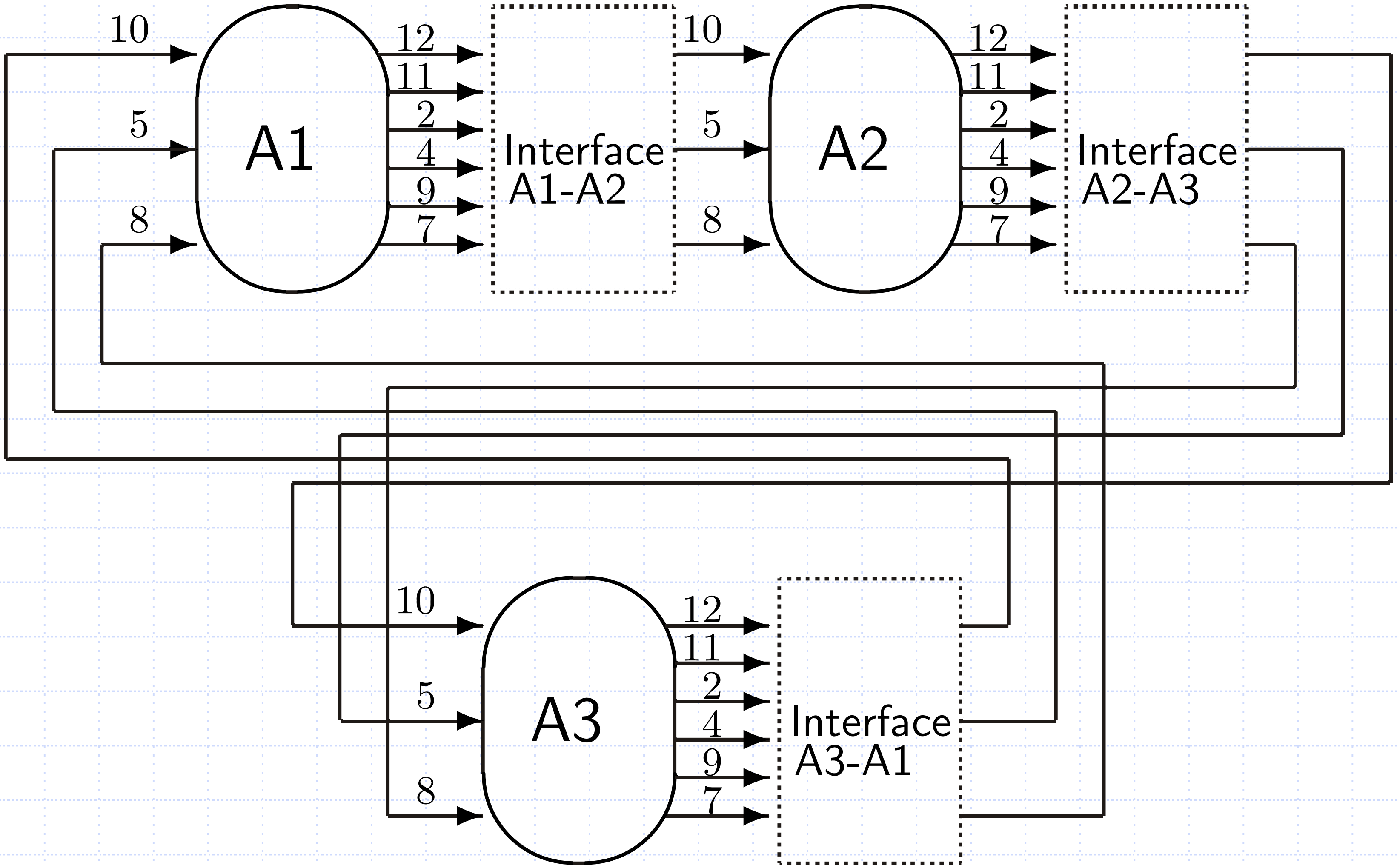
$$\mathbf{X}_{reach} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbb{X}_1 = \mathbf{x}_0$$

Reachability trees for all of the three cases



5. Modelling the agents cooperation



$$
\mathbf{F} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & | & \mathbf{F}_{c_1} \\ \mathbf{0} & \mathbf{F}_2 & \cdots & \mathbf{0} & \mathbf{0} & | & \mathbf{F}_{c_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & | & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{F}_{N_A-1} & \mathbf{0} & | & \mathbf{F}_{c_{N_A-1}} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{F}_{N_A} & | & \mathbf{F}_{c_{N_A}} \end{pmatrix}
$$

$; \mathbf{F}_i,\ \mathbf{G}_i,\ i = 1,\ldots,N_A$

parameters of agents

$$
\mathbf{G} = \begin{pmatrix} \mathbf{G}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{G}_{N_A-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{G}_{N_A} \\ --- & --- & --- & --- & --- \\ \mathbf{G}_{c_1} & \mathbf{G}_{c_2} & \cdots & \mathbf{G}_{c_{N_A-1}} & \mathbf{G}_{c_{N_A}} \end{pmatrix}
$$

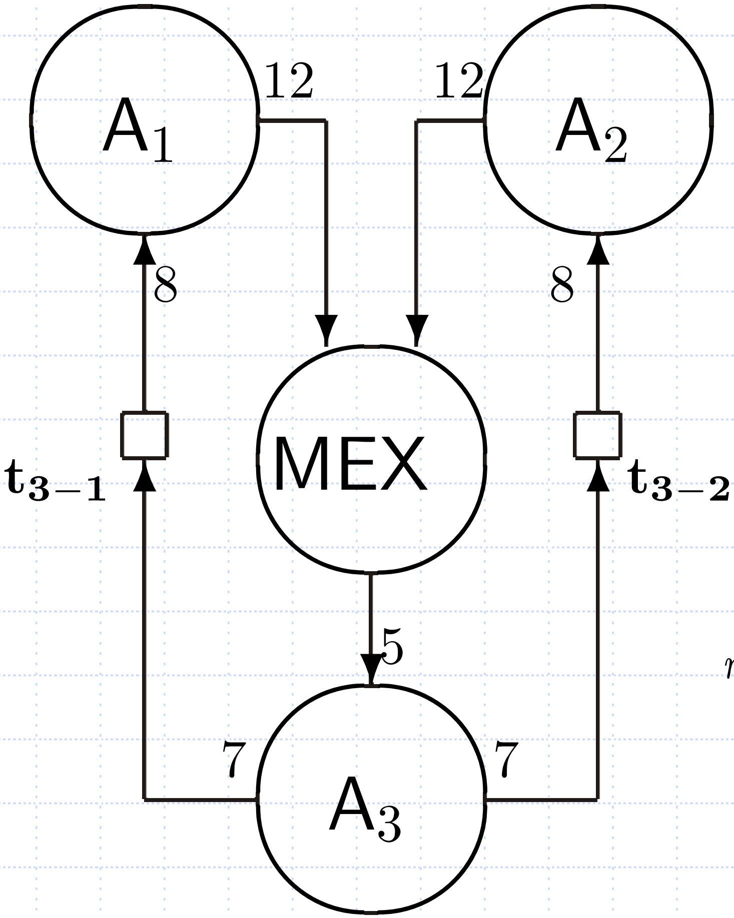
$; \mathbf{F}_{c_i}, \mathbf{G}_{c_i},\ i = 1,\ldots,N_A$

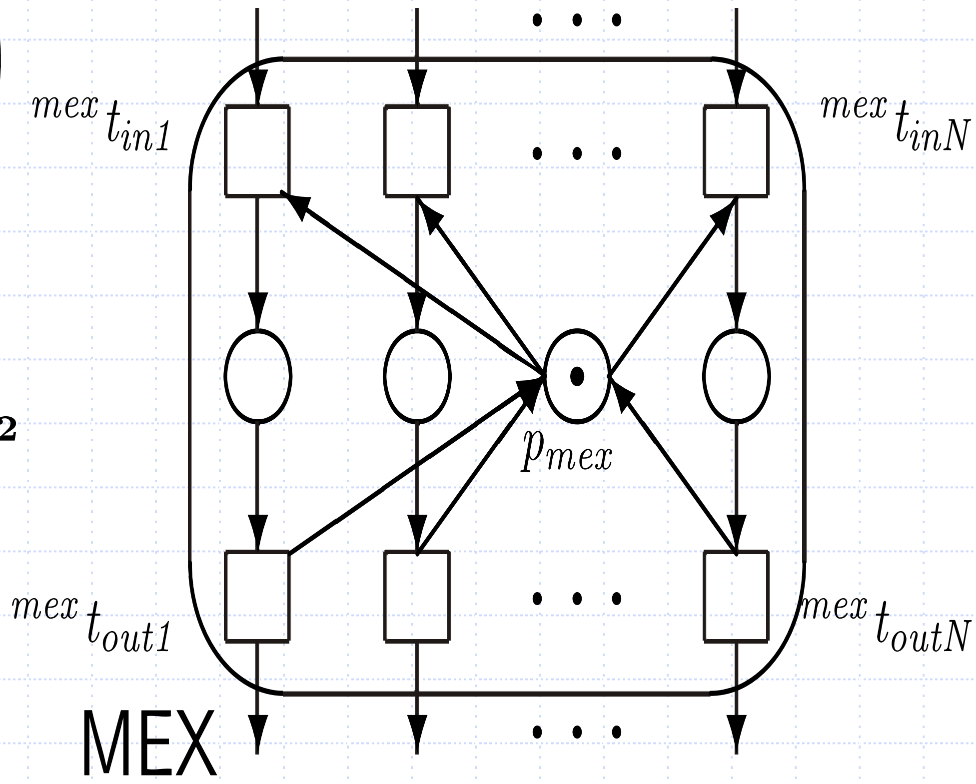
interfaces among agents

$$\mathbf{x}_0 = ({}^{A_1}\mathbf{x}^T{}_0, {}^{A_2}\mathbf{x}^T{}_0, \ldots, {}^{A_{N_A}}\mathbf{x}^T{}_0)^T$$ initial state vector of MAS

$${}^{A_i}\mathbf{x}^T{}_0, \; i = 1,\ldots,N_A$$ initial state vector of A_i

A complex interface





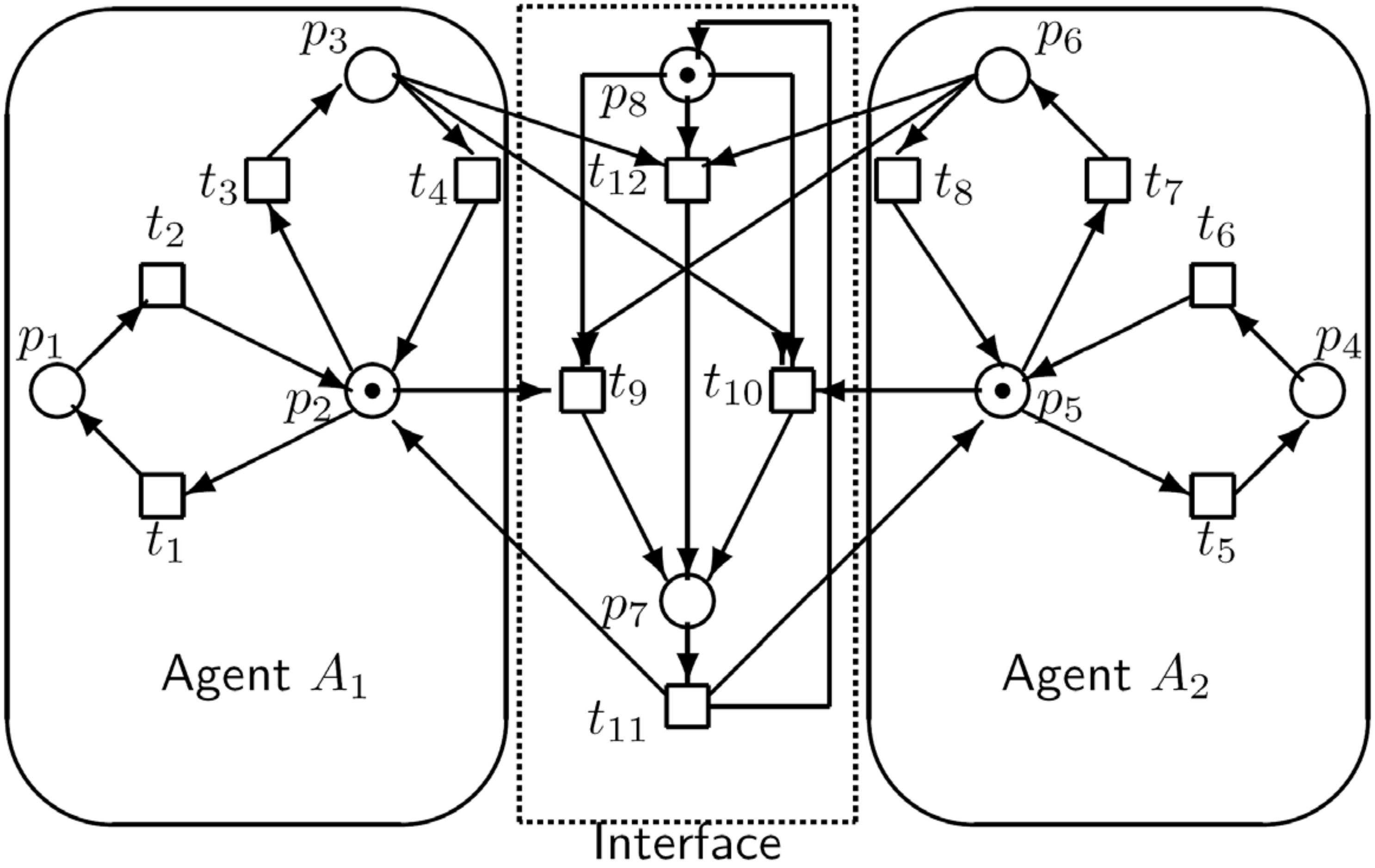
$$\mathbf{F} = \begin{pmatrix} \mathbf{F}_{A_1} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{A_1-Interface} \\ \mathbf{0} & \mathbf{F}_{A_2} & \mathbf{0} & \mathbf{F}_{A_2-Interface} \\ \mathbf{0} & \mathbf{0} & \mathbf{F}_{A_3} & \mathbf{F}_{A_3-Interface} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{Interface} \end{pmatrix} \qquad \mathbf{G}^T = \begin{pmatrix} \mathbf{G}^T_{A_1} & \mathbf{0} & \mathbf{0} & \mathbf{G}^T_{A_1-Interface} \\ \mathbf{0} & \mathbf{G}^T_{A_2} & \mathbf{0} & \mathbf{G}^T_{A_2-Interface} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}^T_{A_3} & \mathbf{G}^T_{A_3-Interface} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}^T_{Interface} \end{pmatrix}$$

system parameters

$$\mathbf{x}_0 = ({}^{A_1}\mathbf{x}^T{}_0\,, {}^{A_2}\mathbf{x}^T{}_0\,, {}^{A_3}\mathbf{x}^T{}_0, {}^{Interface}\mathbf{x}^T{}_0)^T$$

initial state vector

Interface in the form of the PN module



p_1 – *A1* does not want to communicate
p_2 – *A1* is available
p_3 – *A1* wants to communicate
p_4 – *A2* does not want to communicate
p_5 – *A2* is available
p_6 – *A2* wants to communicate
p_7 – communication
p_8 – avilability of the communication channel(s)

t_9 – fires the communication when *A1* is available and *A2* wants to communicate

t_{10} – fires the communication when *A2* is available and *A1* wants to communicate

t_{12} – fires the communication when both *A1* and *A2* want to communicate each other

Because the communication is realized only by transitions, parameters of PN model are:

$$
\mathbf{F} = \left(\begin{array}{cccc|cccc|cccc}
0&1&0&0&0&0&0&0&0&0&0&0\\
1&0&1&0&0&0&0&0&1&0&0&0\\
0&0&0&1&0&0&0&0&0&1&0&1\\
\hline
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&1&0&1&0&0&1&0&0\\
0&0&0&0&0&0&0&1&1&0&0&1\\
\hline
0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&1&1&0&1
\end{array}\right) = \begin{pmatrix}
\mathbf{F}_{A_1} & \emptyset_{(3\times4)} & \mathbf{F}_{A_1\to A_2}\\
\emptyset_{(3\times4)} & \mathbf{F}_{A_2} & \mathbf{F}_{A_2\to A_1}\\
\emptyset_{(2\times4)} & \emptyset_{(2\times4)} & \mathbf{F}_{Ch_{1,2}}
\end{pmatrix}
$$

$$
\mathbf{G}^T = \left(\begin{array}{cccc|cccc|cccc}
1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&1&0&0&0&0&0&0&1&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
\hline
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&1&0&0&1&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
\hline
0&0&0&0&0&0&0&0&1&1&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0
\end{array}\right) = \begin{pmatrix}
\mathbf{G}^T_{A_1} & \emptyset_{(3\times4)} & \mathbf{G}^T_{A_1\to A_2}\\
\emptyset_{(3\times4)} & \mathbf{G}^T_{A_2} & \mathbf{G}^T_{A_2\to A_1}\\
\emptyset_{(2\times4)} & \emptyset_{(2\times4)} & \mathbf{G}^T_{Ch_{1,2}}
\end{pmatrix}
$$

$$
\mathbf{x}_0^T = (0,1,0,0,1,0,0,1)^T
$$

When the communication is realized by means of transitions and places:

$$\mathbf{F} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{F}_{T_1} \\ \mathbf{0} & \mathbf{F}_2 & \cdots & \mathbf{0} & \mathbf{F}_{T_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{F}_{N_A} & \mathbf{F}_{T_{N_A}} \\ \mathbf{F}_{P_1} & \mathbf{F}_{P_2} & \cdots & \mathbf{F}_{P_{N_A}} & \mathbf{F}_{Interface} \end{pmatrix} \qquad \mathbf{G}^T = \begin{pmatrix} \mathbf{G}_1^T & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{G}_{T_1}^T \\ \mathbf{0} & \mathbf{G}_2^T & \cdots & \mathbf{0} & \mathbf{G}_{T_{21}}^T \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{G}_{N_A}^T & \mathbf{G}_{T_{N_A}}^T \\ \mathbf{G}_{P_1}^T & \mathbf{G}_{P_2}^T & \cdots & \mathbf{G}_{P_{N_A}}^T & \mathbf{G}_{Interface}^T \end{pmatrix}$$

indices T_i, $i=1,\ldots,N_A$ denote communication of blocks by means of PN transitions

indices P_i, $i=1,\ldots,N_A$ denote communication of blocks by means of PN places

$\mathbf{F}_{Interface}$, $\mathbf{G}^T_{Interface}$ parameters of the PN block representing the interface

6. Analysis and control synthesis

Hypermodel based on the reachability tree (RT)

$$\mathbf{X}_{k+1} = \mathbf{A}^T_{rt}(k) \,.\, \mathbf{X}_k \;,\; k = 0, 1, \ldots$$

$\mathbf{A}^T_{rt}(k)$ $(N \times N)$ functional adjacency matrix of RT

$$\mathbf{X}_k = ({}^kX_1 \;,\; {}^kX_2 \;, \ldots, {}^kX_N \;)^T \;,\; k = 0, 1, \ldots$$

vicarious state vector

$${}^kX_i = \begin{cases} 1 \text{ if } i = k+1 \\ 0 \;\text{ otherwise} \end{cases} \quad ; i = 1, 2, \ldots, N$$

$\mathbf{x}_0 = \mathbb{X}_1$ real initial state vector

$\mathbf{x}_t \in \{ \mathbb{X}_1, \mathbb{X}_2, \mathbb{X}_3, \ldots , \mathbb{X}_N \}$, e.g. $\mathbb{X}_K = \mathbf{x}_t$ is a feasible terminal state

The vicarious vector $\mathbf{X}_k$ represents the real state vector $\mathbf{x}_k$

The straight-lined RT (SLRT) is generated as:

$$\mathbf{X}_{k+1} = \mathbf{A}^T{}_k \,.\, \mathbf{X}_k \,, \; k = 0, 1, \ldots$$

The backtracking RT (BTRT) is generated as:

$$\mathbf{X}_{k-1} = \mathbf{A}_{k-1} . \mathbf{X}_k, \; k = K, K\text{-}1, \ldots$$

Control synthesis

Store SLRT in the matrix

$$\mathbf{M}_1 = (\mathbf{X}_0, {}^{sl}\{\mathbf{X}_1\}, \ldots, {}^{sl}\{\mathbf{X}_{K-1}\}, {}^{sl}\{\mathbf{X}_K\})$$

and store BTRT in the matrix

$$\mathbf{M}_2 = ({}^{bt}\{\mathbf{X}_0\}, {}^{bt}\{\mathbf{X}_1\}, \ldots, {}^{bt}\{\mathbf{X}_{K-1}\}, \mathbf{X}_K)$$

The intersection yields the space of feasible trajectories

$$\mathbf{M} = \mathbf{M}_1 \cap \mathbf{M}_2 = (\mathbf{X}_0, \{\mathbf{X}_1\}, \ldots, \{\mathbf{X}_{K-1}\}, \mathbf{X}_K)$$

$$\{\mathbf{X}_i\} = {}^{sl}\{\mathbf{X}_i\} \cap {}^{bt}\{\mathbf{X}_i\} = min({}^{sl}\{\mathbf{X}_i\}, {}^{bt}\{\mathbf{X}_i\}), \; i = 0, 1, \ldots, K$$

$${}^{sl}\{\mathbf{X}_0\} = \mathbf{X}_0 \quad , \quad {}^{bt}\{\mathbf{X}_K\} = \mathbf{X}_K$$

Example – three agents cooperation

$$ {}^{A_1}\mathbf{x}_0 = {}^{A_2}\mathbf{x}_0 = (1,1,1,0,0,0,0,0,0,0,0)^T $$

$$ {}^{A_3}\mathbf{x}_0 = (1,1,0,1,0,0,0,0,0,0,0)^T \quad , $$ p_{mex} is active

A_1 resolved the problem P_{A_3} instead A_3:

$$ \{ {}^{A_3}t_2, {}^{A_3}t_7, {}^{F_{C_1}}t, \; {}^{A_1}t_3, {}^{A_1}t_6, {}^{MEX}t_{\text{in}1}, {}^{MEX}t_{\text{out}1} \} $$

A_2 resolved the problem of A_3 :

$$ \{ {}^{A_3}t_2, {}^{A_3}t_7, {}^{F_{C_2}}t, \; {}^{A_2}t_3, {}^{A_2}t_6, {}^{MEX}t_{\text{in}2}, {}^{MEX}t_{\text{out}2} \} $$

Conclusions

- PT PN-based modelling the agents was presented
- RT and/or RG were utilized for analysing the agents cooperation
- the RT-based hypermodel was created
- the control synthesis was performed by means of intersection of SLRT and BTRT