Petri Nets in Discrete-Event and Hybrid Systems Modelling, Analysing, Performance Evaluation and Control

František Čapkovič

Institute of Informatics, Slovak Academy of Sciences

Dúbravská cesta 9, 845 07 Bratislava, Slovakia

E-mail: Frantisek.Capkovic@savba.sk

URL: http://www.ui.sav.sk/home/capkovic/capkhome.htm

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Content

1. Introduction and Preliminaries

Petri nets (PN) in general represent the effective tool for modelling discrete event systems (DES) and hybrid systems (HS).

DES are systems behaviour of which depends on an occurrency of discrete events, i.e. they are discrete in nature (driven by discrete events).

They consist exclusively of discrete variables.

The next state depends on the previous state and the occurrence of a discrete event.

HS consists of both discrete variables and continuous ones.

They are a connection of discrete and continuous subsystems.

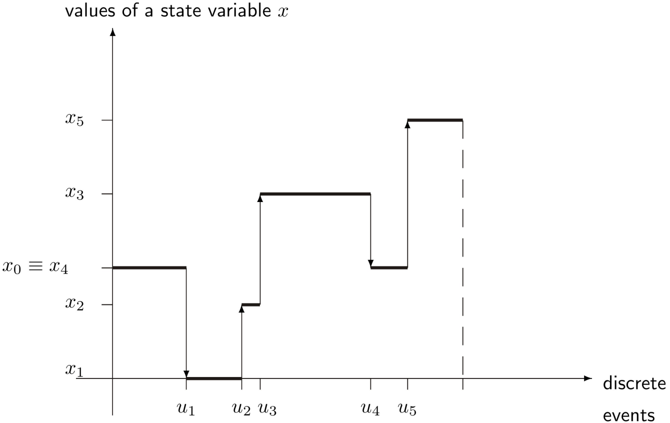


Figure 1. *The development of a state variable x of DES.*

Place/transition PN (P/T PN) are frequently used for modelling DES due to their simple mathematical model given in analytical terms.

Their extended version containing time specifications - timed Petri nets (TPN) - are suitable for modelling the DES behaviour in time and they make possible to obtain the performance evaluation of modelled objects.

Flexible MS (FMS), are the most typical representatives of DES. P/T PN and TPN are very useful for modelling, control and performance evaluation of FMS.

As to their structure P/T PN are bipartite directed graphs

$$\boxed{\langle P, T, F, G \rangle.}$$

The two kinds of nodes are represented by places $p_i \in P,\ i = 1, \ldots, n$, and by the transitions $t_j \in T,\ j = 1, \ldots, m$.

Two kinds of edges (from places to transitions and from transitions to places) are given, respectively, in the form of the sets

$$\boxed{F \subseteq P \times T} \text{ and } \boxed{G \subseteq T \times P}.$$

Next to the structure, P/T PN have also their 'dynamics' formally expressed as

$$\boxed{\langle X, U, \delta, \mathbf{x}_0 \rangle.}$$

Here,

X is the set of the state vectors $\mathbf{x} \in \mathbb{Z}^{n \times 1}$ of the places ($\mathbb{Z}$ symbolizes integers), more precisely $\mathbf{x} \in \mathbb{Z}_{\geq 0}^{n \times 1}$ ($\mathbb{Z}_{\geq 0}$ symbolizes non-negative integers),

U is the set of the state vectors $\mathbf{u} \in \mathbb{Z}^{m \times 1}$ of the transitions, more precisely $\mathbf{u} \in \mathbb{Z}_{\geq 0}^{m \times 1}$ (in general they are the control vectors),

$\boxed{\delta : X \times U \to X}$ is a transition function yielding the new state (marking) vector $\mathbf{x}_{k+1} \in X$ based on both an existing state $\mathbf{x}_k \in X$ and an occurrence of discrete events $\mathbf{u}_k \in U$.

$\mathbf{x}_0$ is the initial state vector of the places

'Dynamics', being the evolution of marking of the P/T PN places, is given by the linear discrete model

$$\boxed{\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{B}.\mathbf{u}_k}$$

restricted by the inequality

$$\boxed{\mathbf{F}.\mathbf{u}_k \leq \mathbf{x}_k,\ k = 0, 1, 2, \ldots N.}$$

Here,

$\mathbf{x}_k = (\sigma_{p_1}, \ldots, \sigma_{p_n})^T$ with $\sigma_{p_i} \in \{0, 1, \ldots, \infty\}$ (i.e. $\sigma_{p_i} \in \mathbb{Z}_{\geq 0}$ (i.e. $\sigma_{p_i} \in \mathbb{N}$ plus 0, where $\mathbb{N}$ symbolizes natural numbers), is the marking vector expressing the state of the marking of the particular places (the number of tokens $n_t \in \mathbb{N}$ inside the places and the empty place expressed by 0)

$\mathbf{u}_k = (\gamma_{t_1}, \ldots, \gamma_{t_m})^T$ with $\gamma_{t_j} \in \{0, 1\}$ is the vector of the states of transitions (either disabled or enabled).

$\mathbf{B} = \mathbf{G}^T - \mathbf{F}$, $\quad \mathbf{B} \in \mathbb{Z}^{n \times m}$, expresses the PN structure.

$\mathbf{F} \in \mathbb{Z}_{\geq 0}^{n \times m}$ (**Pre**), $\mathbf{G}^T \in \mathbb{Z}_{\geq 0}^{n \times m}$ (**Post**) are the incidence matrices of the arcs corresponding, respectively, to the sets F and G.

The places model activities (operations) in FMS, while the transitions model the discrete events (starting/ending the activities).

P/T PN-based model does not depend (explicitly) on time. Their transitions, places, arcs and tokens do not contain any time specifications.

There is only one disadvantage, namely, P/T PN are not able to express explicitly time.

Consequently, to express time relations Timed Petri Nets (TPN) have to be used.

TPN directly yield the marking evolution with respect to (wrt.) time.
In TPN certain time specifications are defined.

Here (in this presentation), the time specifications in TPN are assigned exclusively to the P/T PN transitions as their duration function

$$\boxed{D : \mathcal{T} \to \mathbb{Q}_{\geq 0}}$$

where $\mathbb{Q}_0^+$ symbolizes non-negative rational numbers.
In such a way P/T PN turn to TPN.

In the deterministic case the time specifications are represented by certain time delays of the transitions.

In the non-deterministic cases they express a kind of the probability distribution of timing the transitions - exponential, discrete uniformed, Poisson's, etc.

However, there is no relation between TPN with non-deterministic timing and stochastic Petri nets.

Putting the time specifications into the P/T PN transitions we exactly convert P/T PN into TPN.

A time specification assigned into a transition expresses time necessary for the performance all of operations or activities modelled by the input places of the transition.

Non-deterministic timing can be expressed e.g. by the exponential probability distribution ${}^{e}f_x$ and/or the discrete uniform probability distribution ${}^{u}f_x$ defined as

$$
\boxed{{}^{e}f_x = \begin{cases} \lambda.e^{-\lambda.x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}} ; \quad \boxed{{}^{u}f_x = \begin{cases} 1/(b-a) & \text{if } x \in (a,b) \\ 0 & \text{otherwise} \end{cases} \quad (1)}
$$

Hybrid Petri nets (HPN) in general are another extension of PN.

HPN model HS where discrete and continuous variables coexist.

HPN have two kinds of places and two kinds of transitions - discrete and continuous.

Thus, four kinds of directed arcs occur in HPN - the arcs between:

(i) discrete places and discrete transitions (expressed by $\mathbf{Pre}_{dd} \in \mathbb{Z}_{\geq 0}^{n_d \times m_d}$, $\mathbf{Post}_{dd} \in \mathbb{Z}_{\geq 0}^{n_d \times m_d}$);

(ii) continuous places and continuous transitions ($\mathbf{Pre}_{cc} \in \mathbb{Z}_{\geq 0}^{n_c \times m_c}$, $\mathbf{Post}_{cc} \in \mathbb{Z}_{\geq 0}^{n_c \times m_c}$);

(iii) discrete places and continuous transitions ($\mathbf{Pre}_{dc} \in \mathbb{Z}_{\geq 0}^{n_d \times m_c}$, $\mathbf{Post}_{dc} \in \mathbb{Z}_{\geq 0}^{n_d \times m_c}$);

(iv) continuous places and discrete transitions ($\mathbf{Pre}_{cd} \in \mathbb{Q}_{\geq 0}^{n_c \times m_d}$, $\mathbf{Post}_{cd} \in \mathbb{Q}_{\geq 0}^{n_c \times m_d}$).

The HPN discrete places and transitions handle the discrete tokens, while the HPN continuous places and transitions handle the continuous variables (flows).

Especially, first-order hybrid Petri nets (FOHPN) are frequently used for modelling and control of HS.

FOHPN are a simplified (but mathematically improved) kind of HPN.
In FOHPN

$P = P_d \cup P_c$, where P_d is a set of the discrete places (figured by circles) and P_c is a set of the continuous places (figured by double concentric circles).

$T = T_d \cup T_c$, where T_d is a set of the discrete transitions (figured by rectangles) and T_c is a set of the continuous transitions (figured by double rectangles).

T_d consists of a subset of the immediate (no timed) transitions and/or a subset of the timed transitions (deterministic and/or non-deterministic).

There exist two kinds of marking in FOHPN:
(i) the discrete marking being expressed by tokens in the discrete places;
(ii) the continuous marking being expressed by an amount of a substance (fluid) in the continuous places.

The instantaneous firing speed (IFS) $\boxed{V_j^{min} \leq v_j(\tau) \leq V_j^{max}}$ determining an amount of the substance per time unit in a time instant τ, is assigned to each of the continuous transition T_j.

The marking development of the continuous place $P_i \in P_c$ in time is described by the relation (differential equation)

$$dM_i/d\tau = \sum_{T_j \in T_c} C(P_i,\ T_j).v_j(\tau),$$

where

M_i is the marking of the continuous place P_i

$v_{j\in<1,n_c>}(\tau)$ are entries of the IFS vector $\mathbf{v}(\tau) = (v_1(\tau), \cdots, v_{n_c}(\tau))^T$ in the time τ and $\mathbf{C} = \mathbf{Post}_{cc} - \mathbf{Pre}_{cc}$.

The continuous transition T_j is enabled in the time instant τ if and only if:

(i) its input discrete places $p_k \in P_d$ have the marking $m_k(\tau)$ at least equal to the element $Pre_{dc}(p_k,\ T_j)$ of the incidence matrix $\mathbf{Pre}_{dc}$;

(ii) and all of its input continuous places $P_i \in P_c$ satisfy the condition that their markings $M_i(\tau) \geq 0$ - i.e. the places P_i are filled.

T_j cannot take more fluid from any empty input continuous place than the amount entering the place from its input transitions (principle of the mass conservation).

For the so called well-formed FOHPN holds $\boxed{\mathbf{Pre}_{dc} = \mathbf{Post}_{dc}}$.

Particular approaches utilizing different kinds of Petri nets (defined above) will be illustrated on five case studies:

(i) the DES supervisor synthesis by P/T PN;

(ii) the TPN model of DES;

(iii) FOHPN Model of a Hybrid System;

(iv) FOHPN Model of a Batch Process;

(v) P/T PN-Based Collision Avoiding of 4 Robots

All of them are related to FMS.

2. Agent Cooperation Based on Place Invariants

The real devices, especially the industrial robots and machine tools, even the whole production lines, can be understood to be agents cooperating in FMS. Of course, they are not abstract software agents but material ones.

Each agent can be described by the P/T PN-based model.

The global P/T PN-based model can by composed from the models of the particular agents.

At the global model synthesis we will use the methods of the DES control theory, namely the supervisor synthesis. Then the supervisor becomes an additional agent coordinating the activities of other agents. It is proposed wrt. prescribed rules.

We will use the method of the supervisor synthesis based on the P-invariants of P/T PN which is an extended (generalized) method of the mutual exclusion.

Place Invariants:

What does it mean verbally?

Firings of transitions transform the token distribution of a net, but often respect some global properties of markings.

For example, the total token count of a set of places remains unchanged if the pre-set and the post-set of the transition contain the same number of places of this set.

Place invariants formalize such invariant properties.

If in a set of places the sum of tokens remains unchanged after firing the transition, then this set can define a place invariant.

... Agent Cooperation Based on Place Invariants

P-invariants are the columns of the matrix $\mathbf{W} \in \mathbb{Z}^{n \times n_s}$ computed as

$$\mathbf{W}^T.\mathbf{B} = \mathbf{0} \tag{2}$$

However, invariants can be defined alternatively, as vectors $\mathbf{w}$ satisfying the condition

$$\mathbf{w}^T.\mathbf{x}_k = \mathbf{w}^T.\mathbf{x}_0 \tag{3}$$

for each state vector $\mathbf{x}_k$ reachable from the initial state vector $\mathbf{x}_0$.

Also the Parikh's vector is very important in the process of the supervisor synthesis. To obtain it let us evolve the PN model as follows

$$\begin{aligned} \mathbf{x}_k &= \mathbf{x}_{k-1} + \mathbf{B}.\mathbf{u}_{k-1} = \mathbf{x}_{k-2} + \mathbf{B}.(\mathbf{u}_{k-1} + \mathbf{u}_{k-2}) = \dots \\ &= \mathbf{x}_0 + \mathbf{B}.(\mathbf{u}_0 + \mathbf{u}_1 + \dots + \mathbf{u}_{k-1}) = \mathbf{x}_0 + \mathbf{B}.\mathbf{v} \end{aligned} \tag{4}$$

Here, $\boxed{\mathbf{v} = \sum_{j=0}^{k-1} \mathbf{u}_j}$ is named as the Parikh's vector.

... Agent Cooperation Based on Place Invariants

It yields information on how many times the particular transitions are fired during the evolution of P/T PN from the initial state $\mathbf{x}_0$ to a prescribed terminal state $\mathbf{x}_k$.

Thus, the condition (3) acquires the form

$$\mathbf{w}^T.\mathbf{B}.(\mathbf{u}_0 + \mathbf{u}_1 + \ldots + \mathbf{u}_{k-1}) = \mathbf{w}^T.\mathbf{B}.\mathbf{v} \stackrel{!}{=} \mathbf{0} \tag{5}$$

Because in general $\mathbf{v} \neq \mathbf{0}$, the term $\boxed{\mathbf{w}^T.\mathbf{B} \stackrel{!}{=} \mathbf{0}}$.

This is an alternative definition of the P-invariants.
For more invariants the vector $\mathbf{w}$ acquires the form of the matrix of the invariants $\mathbf{W}$.

After imposing some restrictions on the state vector entries σ_{p_i}, $i = 1, \ldots, n$, in the vector form $\boxed{\mathbf{L}.\mathbf{x} \leq \mathbf{b}}$ and removing the inequality we have

$$\mathbf{L}.\mathbf{x} + \mathbf{x}_s = (\mathbf{L}\ \mathbf{I}_s).\begin{pmatrix} \mathbf{x} \\ \mathbf{x}_s \end{pmatrix} = \mathbf{b} \tag{6}$$

Here,

$\mathbf{L} \in \mathbb{Z}_{\geq 0}^{n_s \times n}$, $\mathbf{b} \in \mathbb{Z}_{\geq 0}^{(n_s \times 1)}$, $\mathbf{x}_s \in \mathbb{Z}_{\geq 0}^{(n_s \times 1)}$ is the vector of slacks and $\mathbf{I}_s \in \mathbb{Z}_{\geq 0}^{(n_s \times n_s)}$ is the identity matrix.

We can force the invariants into the definition (2) in the form as follows

$$(\mathbf{L}\ \mathbf{I}_s).\begin{pmatrix} \mathbf{B} \\ \mathbf{B}_s \end{pmatrix} = \mathbf{0} \tag{7}$$

where

$\mathbf{B}_s = \mathbf{G}_s^T - \mathbf{F}_s$, $\mathbf{B}_s \in \mathbb{Z}^{n_s \times m}$, is the structure of supervisor (till now unknown) to be synthesized.

Hence, $\boxed{\mathbf{B}_s = -\mathbf{L}.\mathbf{B}}$ and the initial state of the supervisor follows from (6) in the form $\boxed{{}^s\mathbf{x}_0 = \mathbf{b} - \mathbf{L}.\mathbf{x}_0}$, $\boxed{{}^s\mathbf{x}_0 \in \mathbb{Z}_{\geq 0}^{n_s \times 1}}$.

Imposing the extended condition

$$\mathbf{L}_p.\mathbf{x} + \mathbf{L}_t.\mathbf{u} + \mathbf{L}_v.\mathbf{v} \leq \mathbf{b} \tag{8}$$

where

$\mathbf{L}_p$ is identical with $\mathbf{L}$ from (7)),
$\mathbf{L}_t \in \mathbb{Z}^{(n_s \times n)}$,
$\mathbf{L}_v \in \mathbb{Z}^{(n_s \times m)}$,

we can synthesized the supervisor with further properties concerning not only the entries of the state vector $\mathbf{x}$ but also the control vector $\mathbf{u}$ and the Parikh's vector $\mathbf{v}$

Namely, when $\boxed{\mathbf{b} - \mathbf{L}_p.\mathbf{x} \geq \mathbf{0}}$ is valid, the supervisor with the following structure and initial state arises:

$$\mathbf{F}_s = \max(\mathbf{0},\ \mathbf{L}_p.\mathbf{B} + \mathbf{L}_v,\ \mathbf{L}_t) \tag{9}$$

$$\mathbf{G}_s^T = \max(\mathbf{0},\ \mathbf{L}_t - \max(\mathbf{0},\ \mathbf{L}_p.\mathbf{B} + \mathbf{L}_v)) - \min(\mathbf{0},\ \mathbf{L}_p.\mathbf{B} + \mathbf{L}_v) \tag{10}$$

$${}^s\mathbf{x}_0 = \mathbf{b} - \mathbf{L}_p.\mathbf{x}_0 - \mathbf{L}_v.\mathbf{v}_0, \tag{11}$$

where

$\mathbf{F}_s \in \mathbb{Z}_{\geq 0}^{n_s \times m}$ and

$\mathbf{G}_s^T \in \mathbb{Z}_{\geq 0}^{n_s \times m}$,

guarantees that constraints are verified for the states resulting from the initial state

$$\boxed{{}^{s}\mathbf{x}_0 = \mathbf{b} - \mathbf{L}_p.\mathbf{x}_0 - \mathbf{L}_v.\mathbf{v}_0}$$

Here, the max(.)/min(.) is the maximum/minimum operator for matrices.

However, the max(.)/min(.) are taken element by element.

3. Case Study 1 - DES Supervisor Synthesis by P/T PN

Let us go to illustrate the supervisor synthesis for P/T PN model of a plant.

Consider five autonomous agents A_i, $i = 1, \ldots, 5$ (e.g. intelligent robots) with the same structure.

The structure of the single agent having two states - 'idle' and 'working' - is given in Fig. 2.

In the group of five agents the states 'idle' are modelled by p_1, p_3, p_5, p_7, p_9 and the states 'working' are modelled by p_2, p_4, p_6, p_8, p_{10}.

It is an analogy with the problem of 5 dining philosophers (defined by Dijkstra).

The robots are situated in a circle.

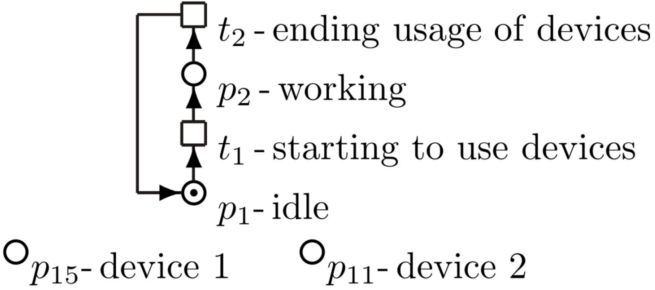


Figure 2. *The PN-based model of the single robot activities.*

... Case Study 1 - DES Supervisor Synthesis by P/T PN

Between two robots a needful identical device is situated.

All of the agents need for their work two such devices.

However, the number of these devices is also only 5.

Each agent has own device but it have to obtain the second device from the left or right neighbour.

It means that the agents have to share the devices.

Formally, the availabilities of the devices are expressed by means of the PN places p_{11}, p_{12}, p_{13}, p_{14}, p_{15} - see Fig. 2, apart from their relations with the robots.

Any robot can take devices only from its neighbours.

The incidence matrices and initial states of the PN models of the robots A_i, $i = 1, \ldots, 5$ are

$$\mathbf{F}_i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \ \mathbf{G}_i^T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \ \mathbf{B}_i = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix};$$

$${}^i\mathbf{x}_0 = (1,\ 0)^T; \ \ i = 1, \ldots, 5$$

The parameters of the PN model of the group of autonomous agents can be expressed as follows

$$\mathbf{F} = \text{blockdiag}(\mathbf{F}_i)_{i=1,5}; \ \mathbf{G} = \text{blockdiag}(\mathbf{G}_i)_{i=1,5}$$

$$\mathbf{x}_0 = ({}^1\mathbf{x}_0^T,\ {}^2\mathbf{x}_0^T,\ {}^3\mathbf{x}_0^T,\ {}^4\mathbf{x}_0^T,\ {}^5\mathbf{x}_0^T)^T$$

The conditions imposed on the autonomous agents are

$$
\begin{aligned}
\sigma_{p_2} + \sigma_{p_4} &\leq 1 \\
\sigma_{p_4} + \sigma_{p_6} &\leq 1 \\
\sigma_{p_6} + \sigma_{p_8} &\leq 1 \\
\sigma_{p_8} + \sigma_{p_{10}} &\leq 1 \\
\sigma_{p_{10}} + \sigma_{p_2} &\leq 1
\end{aligned}
$$

Verbally it means that two adjacent robots (neighbours) must not work simultaneously.

Consequently, the matrix **L** and the vector **b** are as follows

$$\mathbf{L} = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}; \ \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix};$$

$$\mathbf{B}_s = -\mathbf{L}.\mathbf{B}; \ {}^s\mathbf{x}_0 = \mathbf{b} - \mathbf{L}.\mathbf{x}_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\mathbf{B}_s = \begin{pmatrix} -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

$$\mathbf{F}_s = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} ; \mathbf{G}_s^T = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The structural matrices $\mathbf{F}_s$, $\mathbf{G}_s$ of the supervisor give us the structural interconnections between the robots and the devices.

Using the supervisor synthesis the problem was easily resolved.

The PN-based model of the solution - the cooperating agents - is given in Fig. 3.

... Case Study 1 - DES Supervisor Synthesis by P/T PN

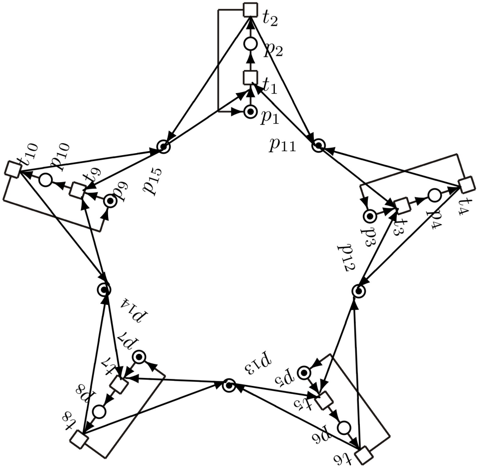


Figure 3. *The PN-based model of the cooperation of 5 agents.*

... Case Study 1 - DES Supervisor Synthesis by P/T PN

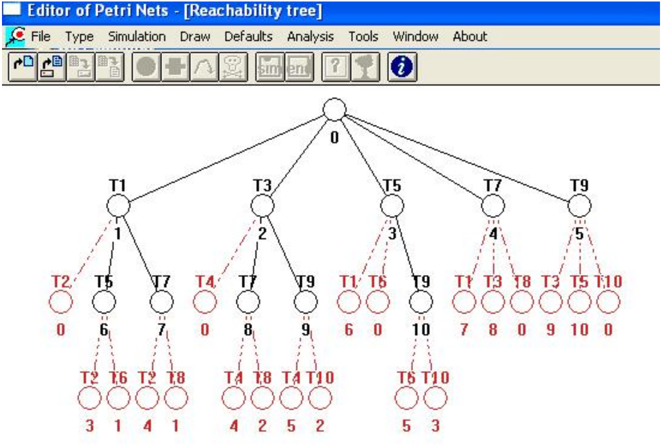


Figure 3a. *The reachability tree (RT) of the PN-based model.*

The RT nodes are the columns of

$$\mathbf{X}_{reach} = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

... Case Study 1 - DES Supervisor Synthesis by P/T PN

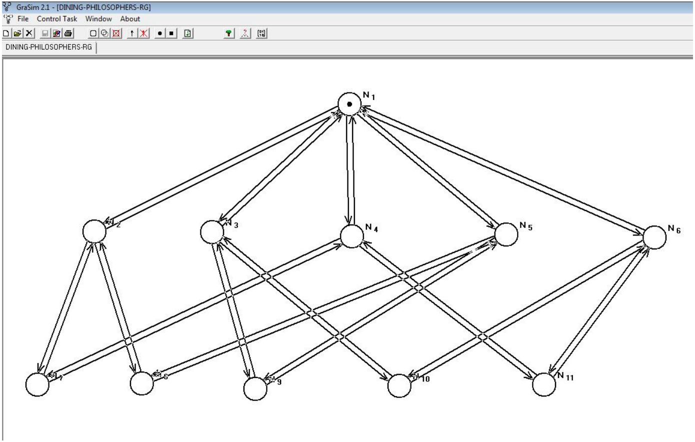


Figure 3b. *The corresponding reachability graph (RG) of the PN-based model.*

4. Case Study 2 - TPN Model of DES

Consider a real flexible manufacturing system consisting of four machines M1, . . ., M4.
Three types of products P1, . . ., P3 are produced by means of the machines.

A few of the machines, but not always all of them, are involved in the machining of each type of product:

> To produce P1 all machines are involved in the order M1, M2, M3, M4;
> to produce P2 three machines are involved in the order M1, M4, M3;
> to produce P3 three machines are involved in the order M1, M2, M4.

There is the following demand: for P1 and P2 two products P3 have to be produced.

Denote these two products as P31 and P32.

... Case Study 2 - TPN Model of DES

Let us use the agent-based approach.

To model the system, define eight agents A1 - A8.

The agents A1 - A4 express, respectively, the working cycles of the machines M1 - M4.

The agents A5 - A8 express, respectively, the cyclic processes of machining the parts P1, P2, P31, P32.

The P/T PN models of the particular agents are apparent from the *latticed* formation given in Fig. 4.

... Case Study 2 - TPN Model of DES

For both P31 and P32 the sequence of machines is the same, i.e. M1, M2, M4, of course.

The technological process is roughly expressed in Tab. 1.

Table 1. *The scheme of the technological process*

order of using the machines	P1	P2	P31	P32
1	M1	M1	M1	M1
2	M2	-	M2	M2
3	M3	M4	-	-
4	M4	M3	M4	M4

The columns prescribe in which order the particular machines are used at machining the particular products.

... Case Study 2 - TPN Model of DES

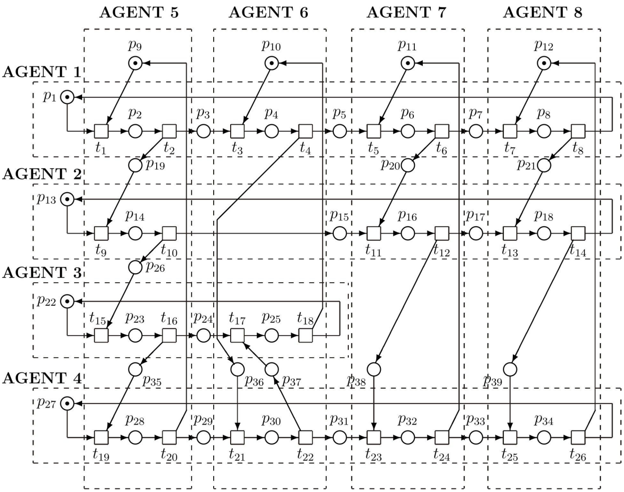


Figure 4. *The global model of FMS consisting of the cooperating agents modelled by P/T PN based models.*

Here, the availability of the machines M1, M2, M3, M4 is represented, respectively, by means of the active (i.e. with tokens) PN places $p_1, p_{13}, p_{22}, p_{27}$.

The availability of the raw material or semiproducts for the machines M1, M2, M3, M4 is expressed, respectively, by means of the active PN places $p_9, p_{10}, p_{11}, p_{12}$.

In the *lattice* P/T PN model of the global system, the *horizontal* agents A1 - A4 express, respectively, the working cycles of the corresponding machines M1 - M4.

The *vertical* agents A5 - A8 express, respectively, the working cycles for producing the products P1, P2, P31, P32.

... Case Study 2 - TPN Model of DES

The classical analysis of the system trajectories (by means of the P/T PN model) is practically impossible because of the too large state space. Namely, the reachability tree (RT) has 406 nodes that are intricately linked.

Assign time to the transitions of the global P/T PN model in order to obtain TPN model. Using nondeterministic timing with discrete uniform probability distribution (1) - ${}^{u}f_x$ - where a, b for particular transitions are the entries of the following vectors

$$\begin{aligned} \mathbf{a} &= (.2, .9, .2, .9, .2, .9, .2, .9, .2, .9, .2, 1.85, .2, 1.85, \\ & .2, 2.75, .2, 1.85, .2, 2.75, .2, .9, .2, .9, .2, .9) \\ \mathbf{b} &= (.3, 1.1, .3, 1.1, .3, 1.1, .3, 1.1, .3, 1.1, .3, 2.15, .3, \\ & 2.15, .3, 3.25, .3, 2.15, .3, 3.75, .3, 1.1, .3, 1.1, .3, \\ & 1.1), \end{aligned} \tag{12}$$

the TPN model arises.

... Case Study 2 - TPN Model of DES

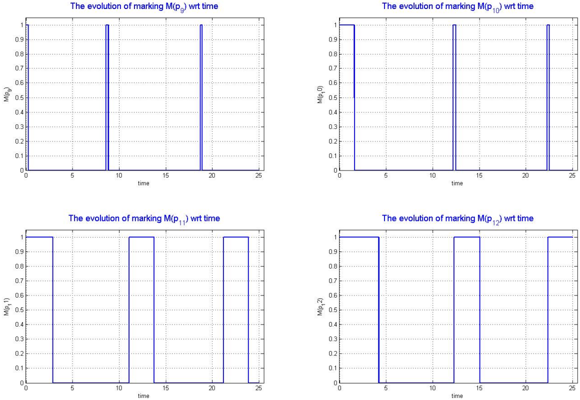


Figure 5. *The supply of the machines M1 - M4 (of TPN based model of FMS) by the input raw material in time - places* $p_9 - p_{12}$

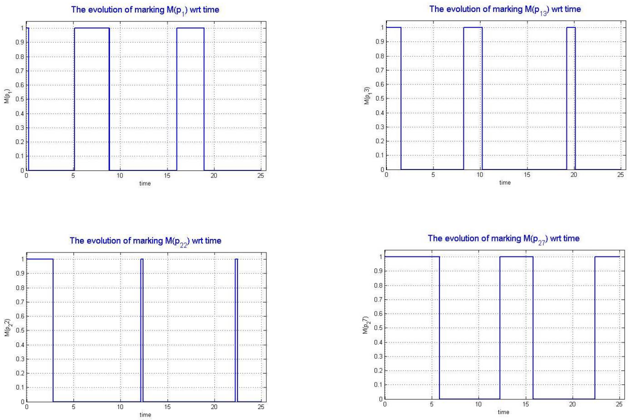


Figure 6. *The operating time of the machines M1 - M4 (represented by the places p_1, p_{13}, p_{22}, p_{27}) machining the raw material and/or semiproducts to produce the final shape of the products*

... Case Study 2 - TPN Model of DES

By means of simulation employing the TPN model in the tool HYPENS for MATLAB, the results given in Fig. 5 and in Fig. 6, were found.

In Fig. 5, the fragment of dynamic behaviour of the *vertical* agents A5 - A8, which supply A1 with semiproducts, is displayed.

In Fig. 6, the fragment of dynamic behaviour of the *horizontal* agents A1 - A4, which start to process the semiproducts, is displayed.

Of course, the simulation process yields the dynamic behaviour all of the TPN places.

But because of the limited space it is impossible to introduced here all of them.

5. Case Study 3 - FOHPN Model of a Hybrid System

Consider a real manufacturing system consisting of a production line of a small enterprise producing plastic bags from granulate prepared from waste plastic by a recycling procedure.

The line produces large double foil bales of a prescribed weight.
The double foil has a form of a sleeve.

Let us use FOHPN for modelling the line having hybrid (i.e. continuous/discrete) character.

The scheme of the line is given in Fig. 7.

To distinguish continuous and discrete places as well as continuous and discrete transitions, the continuous and discrete items are denoted by capital and lowercase letters, respectively.

... Case Study 3 - FOHPN Model of a Hybrid System

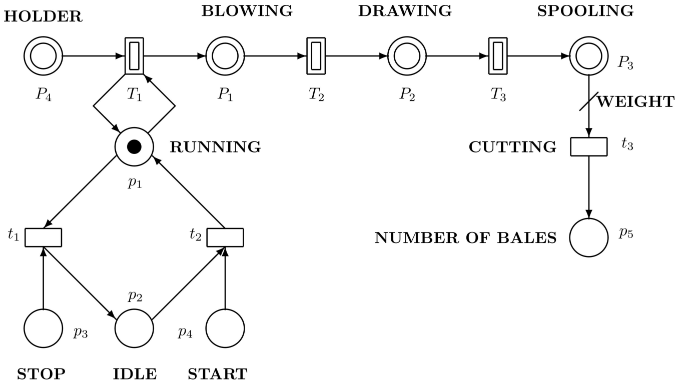


Figure 7. *The rough FOHPN-based model of the single production line.*

... Case Study 3 - FOHPN Model of a Hybrid System

The granulate is located in a holder represented by the continuous place $P4$.

From there, the granulate flows through continuous transition $T1$ to the blowing machine (blower) represented by the continuous place $P1$ where a paste is prepared by help of heat and then a big bubble is blown (to enable producing the double foil).

Subsequently, after a time to avoid double foil re-joining, the double foil is drawn into the required width and thickness on drawing line $P2$ and proceeds to spooling machine $P3$ where the bales of a required weight are prepared.

Here, having achieved the required weight, the double foil is cut off the completed bale is removed and new bale starts to be spooled on a new spool.

... Case Study 3 - FOHPN Model of a Hybrid System

The completed bale proceeds to the buffer or to another production line (rolling line) where rolls of bags are produced.
(The factory produces bags for customers on the made-to-order job principle).
There, the double foil bale is gradually unrolled, welds corresponding to the bag length and the belt of bags is rolled into rolls with a uniform number of bags in each.

Marking of the discrete place p_5 expresses the number of bales produced by the line.

This place can be understood to be the buffer. The discrete places p_3, p_4 manage transitions of the line from idle (p_2) to running (p_1) state and vice versa.

More detailed FOHPN model is given in Fig. 8.

... Case Study 3 - FOHPN Model of a Hybrid System

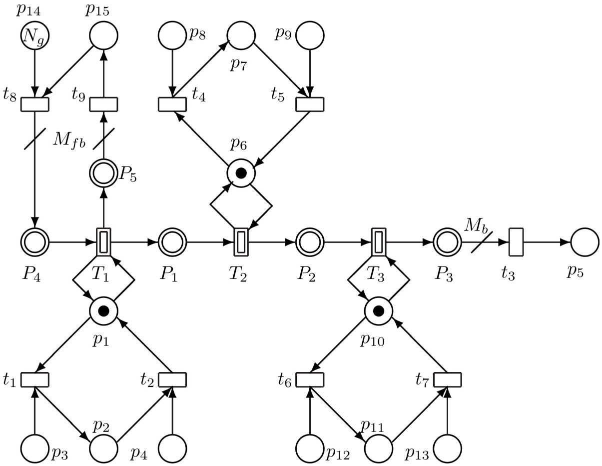


Figure 8. *The detailed FOHPN-based model of the single production line.*

The parameters of the FOHPN model are given as follows

$$\mathbf{Pre}_{cc} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \; \mathbf{Post}_{cc} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{Pre}_{cd} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_b & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{fb} \end{pmatrix};$$

$$\mathbf{Post}_{cd} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{fb} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{Pre}_{dd} = \begin{pmatrix} 1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1&0 \end{pmatrix} ; \ \mathbf{Post}_{dd} = \begin{pmatrix} 0&1&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$\mathbf{Pre}_{dc}^{T} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \equiv \mathbf{Post}_{dc}^{T}$$

The initial marking of the continuous places is

$$\boxed{\mathbf{M}_c = (0,\, 0,\, 0,\, M_{gr},\, 0)^T}$$

where M_{gr} is the initial amount of the granulate in P_4.

The initial marking of the discrete places is

$$\boxed{\mathbf{m}_d = (1,\, 0,\, 1,\, 1,\, 0,\, 1,\, 0,\, 1,\, 1,\, 1,\, 0,\, 1,\, 1,\, N_g,\, 0)^T}$$

where N_g is the number of the batches of the granulate to be added during the production.

... Case Study 3 - FOHPN Model of a Hybrid System

The other parameters in the above matrices are

M_b is the prescribed weight of the bale
M_{fb} denotes the added amount of the granulate in one batch.

Firing speeds of the continuous transitions $v_j(\tau)$, $j = 1, 2, 3$ are, respectively, from the intervals $[V_j^{min}, V_j^{max}]$.

The discrete transitions are considered to be deterministic without any delay or with a transport delays mentioned above.

Using the Matlab simulation tool HYPENS with the following values of the parameters:

... Case Study 3 - FOHPN Model of a Hybrid System

structural parameters $M_b = 270$, $M_{fb} = 3750$,
initial markings with $M_{gr} = 5000$, $N_g = 4$,
limits of intervals for the firing speeds being $V_j^{min} = 0$, $j = 1, 2, 3$, $V_1^{max} = 1.8$, $V_2^{max} = 1.5$ and $V_3^{max} = 1.4$ and
delays of discrete transitions being the entries of the vector

$$\boxed{(0,\ 0,\ 0,\ 0.01,\ 125,\ 0.01,\ 300,\ 0,\ 0)^T}$$

we obtain the behaviour of the simulated line.

It is illustrated in Fig. 9 and Fig. 10.

In the left part of Fig. 9 the course of flows is shown during the time segment when the granules are added four times while the right part displays the zoomed detail.

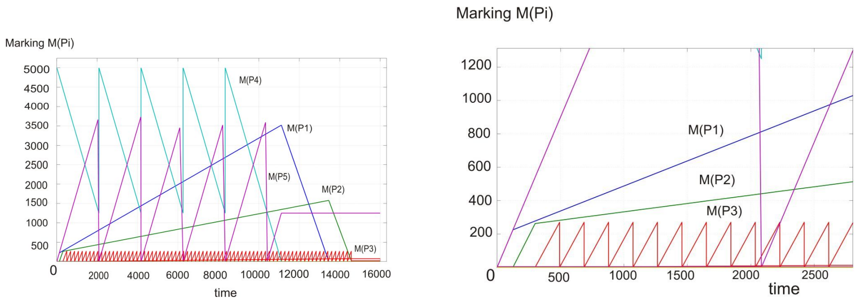


Figure 9. *The dynamics behaviour of the line material flows in the common scale (left picture) and the zoomed detail in order to see the transport delays better (right picture)*

... Case Study 3 - FOHPN Model of a Hybrid System

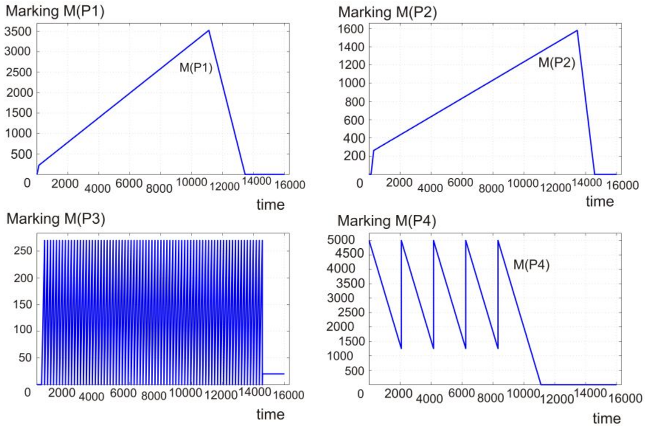


Figure 10. *The dynamics behaviour of the material flows of the production line in their individual scales - i.e. the marking evolution of the continuous places P_1 - P_4 in time*

Consider 4 such production lines feeding 2 rolling machines.

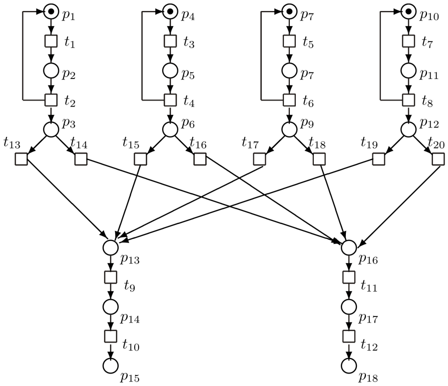


Figure 10a *The Petri net-based model of the rough conception of the supposed cooperation of the production lines*

Four upper lines produce the plastic double foil from the granulate prepared from the waste plastic. Here, only the cooperation of the lines will be discussed.

Two lower lines produce rolls of plastic bags from the double foil. In **Fig. 10a** only a rough scheme of the cooperation of two groups of lines is displayed

$\{p_1, p_4, p_7, p_{10}\}$ ... the continuous production of the foil

$\{p_2, p_5, p_8, p_{11}\}$ ... cutting a bales of the foil with a determined weight and delivering the bale to the buffers $\{p_3, p_6, p_9, p_{12}\}$

$\{p_{13}, p_{16}\}$ ... taking bales of double foil from the buffers

$\{p_{14}, p_{17}\}$ ... rolling the belt into rolls of a prescribed length (prescribed number of bags) and

$\{p_{15}, p_{18}\}$ ... buffers of the rolls

At forming the rules defining the mutual cooperation of the lines we have to respect the facts as follows:

1. any bale of the foil from output buffers of the four foil production lines can enter only one of the two rolling machines;
2. only one bale can enter any rolling machine;
3. next bale can enter the rolling machines after finishing the rolling process

While (i), (ii) mean that the transition functions of the PN transitions $t_{13} - t_{20}$ have to satisfy

$$\gamma_{t_{13}} + \gamma_{t_{15}} + \gamma_{t_{17}} + \gamma_{t_{19}} \leq 1 \tag{13}$$

$$\gamma_{t_{14}} + \gamma_{t_{16}} + \gamma_{t_{18}} + \gamma_{t_{20}} \leq 1 \tag{14}$$

(iii) means that the places $p_{13} - p_{14}$, $p_{16} - p_{17}$ have to meet

$$\sigma_{p_{13}} + \sigma_{p_{14}} \leq 1 \tag{15}$$

$$\sigma_{p_{16}} + \sigma_{p_{17}} \leq 1 \tag{16}$$

Here, $\mathbf{L}_v = \mathbf{0}$, $\mathbf{L} \neq \mathbf{0}$ and $\mathbf{L}_t \neq \mathbf{0}$.
Consequently,

$$\mathbf{F}_s = \max(\mathbf{0}, \mathbf{L}_p.\mathbf{B}, \mathbf{L}_t) \quad (17)$$

$$\mathbf{G}_s^T = \max(\mathbf{0}, \mathbf{L}_t - \max(\mathbf{0}, \mathbf{L}_p.\mathbf{B})) - \min(\mathbf{0}, \mathbf{L}_p.\mathbf{B}) \quad (18)$$

$$^s\mathbf{x}_0 = \mathbf{b} - \mathbf{L}_p.\mathbf{x}_0 \quad (19)$$

Because

$$\mathbf{L_p} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\mathbf{L}_t = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

the supervisor with the structure

$$\mathbf{F}_s = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathbf{G}_s^T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and the initial state

$${}^{s}\mathbf{x}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

was found.

Then, the PN model of cooperating lines is given in Fig. 10b.

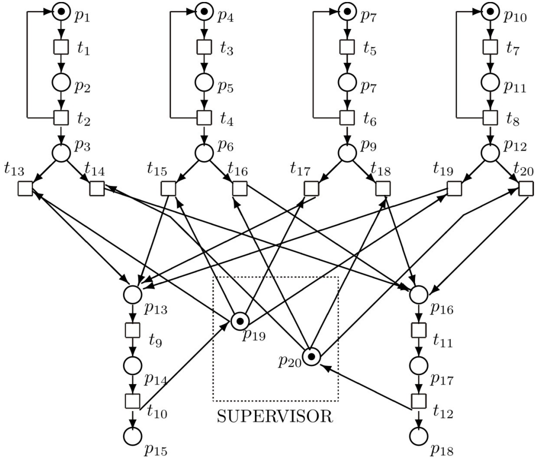


Figure 10b *The Petri net-based model of the supervised cooperation of the production lines*

For a mass production of hundreds (even thousands) parts the P/T PN model as well as the corresponding TPN model may be quite large.

Consequently, the handling such models in the simulation process may be too tedious, even impossible.

Therefore, let us utilize the FOHPN model.

Here, the parts do not arrive individually.

They arrive in batches of the parts to be machined.

Consider the working station producing two kinds of parts π_1 and π_2.

... Case Study 4 - FOHPN Model of a Batch Process

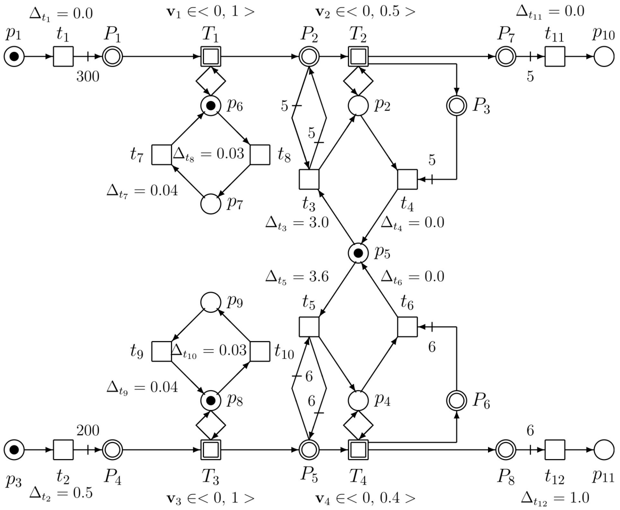


Figure 11. *The FOHPN model of the manufacturing system.*

The upper producing line (L_1) in Fig. 11 produces the parts π_1 while the lower line (L_2) produces the parts π_2.

The machine tool is represented by the discrete place p_5.

The incidence matrices of the FOHPN model are the following

$$\mathbf{Pre}_{cc} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \mathbf{Post}_{cc} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \mathbf{Pre}_{dc} = \mathbf{Post}_{dc} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

... Case Study 4 - FOHPN Model of a Batch Process

$$\mathbf{Pre}_{cd} = \begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&5&0&0&0&0&0&0&0&0&0\\ 0&0&0&5&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&6&0&0&0&0&0&0&0\\ 0&0&0&0&0&6&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&5&0\\ 0&0&0&0&0&0&0&0&0&0&0&6 \end{pmatrix};\ \mathbf{Post}_{cd} = \begin{pmatrix} 300&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&5&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&200&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&6&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$\mathbf{Pre}_{dd} = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&1&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix};\ \mathbf{Post}_{dd} = \begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

In our case in the batches of 300 parts in L_1 and 200 parts in L_2.

These batches are taken into account in Fig. 11 by means of the weights of the input arcs of the continuous places P_1 (in $L1$) and P_4 (in $L2$).

After the arrival of a batch, the parts are downloaded into buffers (P_2 and P_5) at the speed $v_1 \in < 0, 1 >$ (in the number of parts per the time unit) of T_1 and $v_3 \in < 0, 0.5 >$ of T_3.

The speeds of the T_2 and T_4 are set up, respectively, on the values $v_2 \in < 0, 0.5 >$ and $v_4 \in < 0, 0.4 >$.

The processing of the particular parts does not start immediately. It waits until at least 5 parts of type π_1 (see the weight of the arc $P_2 \to t_3$) and 6 parts of type π_2 (see the weight of the arc $P_5 \to t_5$) will be stored, respectively, in the corresponding buffers P_2 and P_5.

Then, the uploading of the machine can be done once more.
Suppose that the machining of the parts π_1 takes 3.0 time units while the machining of the parts π_2 takes 3.6 time units.
Therefore, $\Delta_{t_3} = 3.0$ while $\Delta_{t_5} = 3.6$.

Other discrete transitions have the delays set up as follows:

$\Delta_{t_1} = 0.0$ as well as Δ_{t_4}, Δ_{t_6} and $\Delta_{t_{11}}$;
$\Delta_{t_2} = 10.0$;
$\Delta_{t_7} = 0.04$ as well as Δ_{t_9};
$\Delta_{t_8} = 0.03$ as well as $\Delta_{t_{10}}$;
$\Delta_{t_{12}} = 1.0$.

After processing all of the parts in the batch, the machine will be prepared to process another batch. The machined pieces are removed in the batches having the input size (i.e. 5 and 6 respectively).

... Case Study 4 - FOHPN Model of a Batch Process

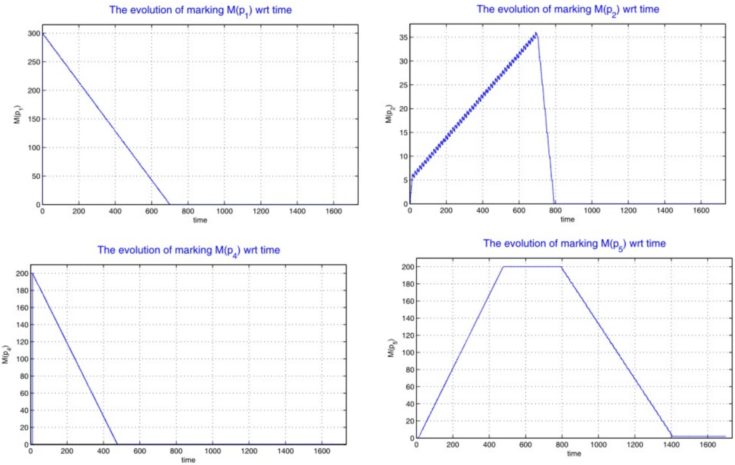


Figure 12a. *The simulation results for the deterministic timing the FOHPN discrete transitions. In the particular pictures the marking evolution of the FOHPN continuous places* P_1, P_2, P_4, P_5 *are displayed.*

... Case Study 4 - FOHPN Model of a Batch Process

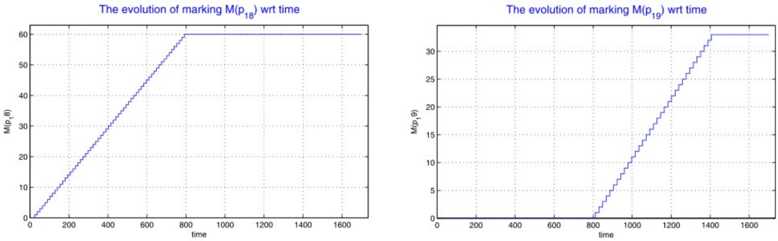


Figure 12b. *The simulation results for the deterministic timing the FOHPN discrete transitions. In the particular pictures the marking evolution of the FOHPN discrete places* p_{10}, p_{11} *(being, respectively, the stacks of the finished parts* π_1, π_2*) wrt. time are displayed.*

... Case Study 4 - FOHPN Model of a Batch Process

The fired continuous transitions T_1, T_2, T_3 and T_4 approximate the discrete movement of the parts by the continuous flows.

Consequently, the model dimensionality and especially the simulation time are extensively reduced.

The replacement of the discrete PN model by the FOHPN model makes the analysis of the real FMS simplier.

Thus, the FOHPN models, primarily determined for modelling the systems hybrid by nature (HS), can be used also for modelling some of the pure DES.

As it was demonstrated, in case of the mass production FOHPN can be exploited also for modelling of the batch processes.

EXPERIMENT

Try now (theoretically) to test whether the alternative processing of different batches is possible or not. Use the non-deterministic model

Consider the exponential probability distribution - ${}^{e}f_x$ (1) - for timing the discrete transitions with

$$\boxed{\lambda_i \in (0.001, 1.0, 0.3, 0.001, 0.36, 0.001, 0.04, 0.03, 0.04, 0.03, 0.001, 0.1)}.$$

Other parameters of the model are the same like before.

The simulation results of the experiment are introduced in Fig. 13a, 13b.

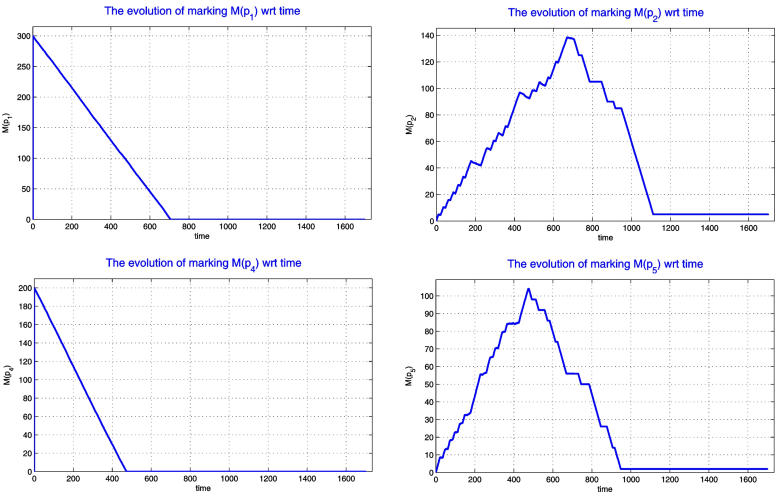
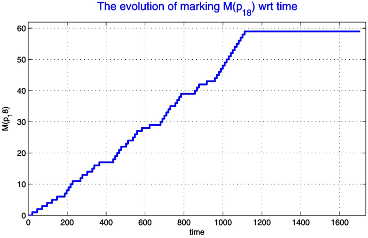

The evolution of marking M(p1) wrt time
M(p1)
time
The evolution of marking M(p2) wrt time
M(p2)
time
The evolution of marking M(p4) wrt time
M(p4)
time
The evolution of marking M(p5) wrt time
M(p5)
time

Figure 13a

... Case Study 4 - FOHPN Model of a Batch Process



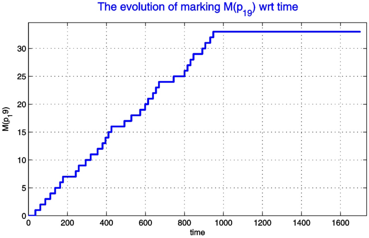


Figure 13b The simulation results for the exponential probability distribution of timing the FOHPN discrete transitions. In the particular pictures the marking evolution of the FOHPN continuous places P_1, P_2, P_4, P_5 and the discrete places p_{10}, p_{11} (being, respectively, the stacks of the finished parts π_1, π_2) wrt. time is displayed.

... Case Study 4 - FOHPN Model of a Batch Process

They show that *theoretically* the machine is able to alternate the batches from both lines.

It is apparent from the comparison of the courses of corresponding markings in Fig. 12a,b and Fig. 13a,b.

While in deterministic case the markings fluently accrue/descend, in the case of the exponential probability distribution they accrue/descend roughly.

It is caused by alternating the lines.

However, global production time of the lines in non-deterministic case is shorter by about 30 percents compared to deterministic one.

But, applicability has to be verified in practice.

Consider four robots R_i, $i = 1, \ldots, 4$ working in the common working space (WS). They are symbolically displayed in Fig. 13.

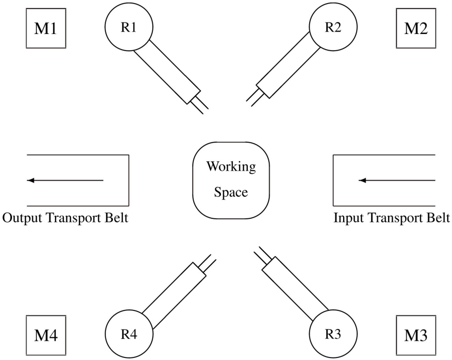


Figure 14: The scheme of the cooperation of 4 robots.

... Case Study 5

Each robot R_i takes a raw material (or semi-product) from the Input Transport Belt (ITB) and inserts it to the corresponding machine M_i.

After machining in M_i the robot R_i takes the finished part from M_i and put it on the Output Transport Belt (OTB).

WS is limited, because the robots obstruct each other.

Namely, they have to pass the WS *criss-cross* to reach ITB and OTB.

Therefore, it is necessary to resolve the potential conflicts in order to avoid any collision.

In the opposite case a crash of the robots can occur and the robots can be damaged or completely destroyed. Moreover, the life of humans being near by, may be endangered.

First of all the P/T PN model of the system expressing the structure of the robotic cell has to be built.

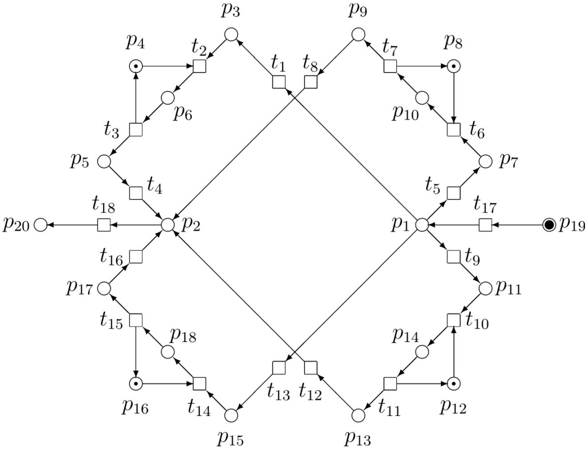


Figure 15: The P/T PN model of the non-supervised activity of 4 robots.

The P/T PN model consists of 4 sub-nets with the same structure where

$$\mathbf{F}_i = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} ; \ \mathbf{G}_i^T = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} ; \ i = 1, \dots, 4$$

The activities of the robot R_1 are modelled by the places

p_1 - taking the raw material from ITB

p_3 - inserting it to M_1

p_5 - unloading the finished part from M_1 and

p_2 - putting the finished part to OTB.

Analogically, the activities of the robot R_2 are modelled by the places

p_1, p_2

p_7 - inserting the raw material to M_2

p_9 - unloading the finished part from M_2.

Likewise, the activities of the robot R_3 are modelled by the places

p_1 and p_2

p_{11} - inserting the raw material to M_3

p_{13} - unloading the finished part from M_3.

Finally, the activities of the robot R_4 are modelled by the places

p_1 and p_2

p_{15} - inserting the raw material to M_4

p_{17} - unloading the finished part from M_4.

The machine tool M_1 is modelled by the places

p_4 (idle, i.e. available) and

p_6 (working).

Analogically, M_2 is modelled by

p_8 (idle) and

p_{10} (working).

Likewise, M_3 is modelled by

p_{12} (idle) and

p_{14} (working).

Finally, M_4 is modelled by

p_{16} (idle) and

p_{18} (working).

The place p_{19} represents ITB. The big token placed inside p_{19} expresses an amount of pieces of the raw material arriving by ITB to the robotic cell from a store.

The place p_{20} models OTB, where the finished parts are put and go away from the robotic cell to another store.

The structure of the P/T PN model given by the incidence matrices and its initial state are the following

$$
\mathbf{F} = \begin{pmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

... Case Study 5

... P/T PN-based modelling the cell

$$
\mathbf{G}^T = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
$$

$$
\mathbf{x}_0^T = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 4 & 0 \end{pmatrix}
$$

However, the model given in Fig. 14 is not yet applicable.

There are two reasons for this:

(i) it does not solve the problem of the collisions

(ii) moreover, it has too large state space (too many reachable states), namely 3338.

Consequently, it is necessary to synthesize a supervisor based on P-invariants having the ability to resolve the conflicts and reduce the state space.

The principal conditions for the elimination of the collisions have to be defined as follows

$$\sigma_{p_1} + \sigma_{p_3} + \sigma_{p_7} + \sigma_{p_{11}} + \sigma_{p_{15}} \leq 1 \tag{20}$$

$$\sigma_{p_2} + \sigma_{p_5} + \sigma_{p_9} + \sigma_{p_{13}} + \sigma_{p_{17}} \leq 1 \tag{21}$$

where σ_{p_i}, $i = 1, 2 \dots, 20$ represent the marking of the places p_i - i.e. the number of tokens placed inside them.

It means that only one place from those competing in (20) and only one place from those competing in (21) can be active.

Utilizing the approach based on the P-invariants of P/T PN we can acquire the supervisor structure and initial state as follows.

Namely, starting from

$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$
$$\mathbf{b} = \begin{pmatrix} 1 & 1 \end{pmatrix}^T \qquad (22)$$

corresponding to (20), (21), we obtain the following structure and initial state of the supervisor (based on the P-invariants)

$$\mathbf{B}_s = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}$$
$${}^s\mathbf{x}_0^T = \begin{pmatrix} 1 & 1 \end{pmatrix} \qquad (23)$$

The P/T PN model of the supervised robot cooperation is in Fig. 15.

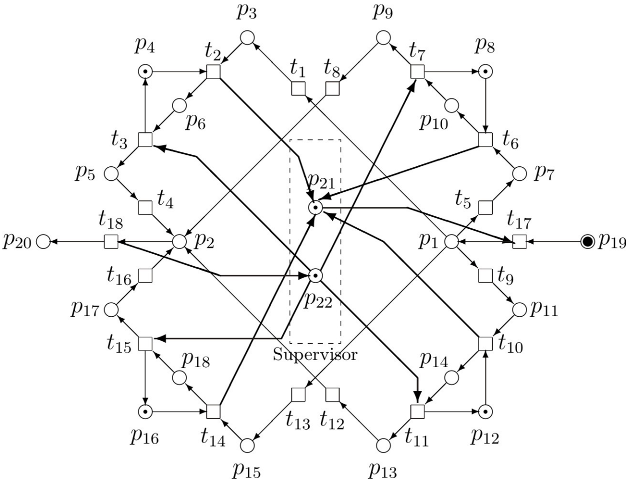


Figure 16: The P/T PN model of the supervised cooperation of 4 robots.

The supervisor places $\boxed{p_{21},\ p_{22}}$ are framed by the dashed rectangle while their interconnections with the transitions of the original model are expressed by the thick edges.

These interconnections can be expressed by means of the following incidence matrices obtained by the decomposition of $\mathbf{B}_s = \mathbf{G}_s^T - \mathbf{F}_s$

$$\mathbf{F}_s = \begin{pmatrix} 0000000000000000010 \\ 0010001000100001000 \end{pmatrix} \quad (24)$$

$$\mathbf{G}_s^T = \begin{pmatrix} 0100010001000010000 \\ 0000000000000000001 \end{pmatrix} \quad (25)$$

The number of states (i.e. the RG nodes) of the supervised model is 793.

The conflicts were eliminated and the number of states was reduced more than 4 times (about 4.2).

... Case Study 5

Setting priorities

How to determine priorities among the robots?

Such a question obtrudes.
Performing the following steps:

1) putting $\mathbf{L} = \mathbf{0}$, $\mathbf{L}_t = \mathbf{0}$, $\mathbf{L}_v \neq \mathbf{0}$ and
2) expressing the priorities by means of the $\mathbf{v}$ (Parikh's vector) entries

we obtain the following structure and initial state of the additional supervisor

$$\mathbf{F}_s = \max(\mathbf{0}, \mathbf{L}_v) \tag{26}$$

$$\mathbf{G}_s^T = \max(\mathbf{0}, (-\max(\mathbf{0}, \mathbf{L}_v))) - \min(\mathbf{0}, \mathbf{L}_v) \tag{27}$$

$${}^s\mathbf{x}_0 = \mathbf{b} - \mathbf{L}_v.\mathbf{v}_0 \tag{28}$$

The supervisor ensures the prescribed priorities.

Consider e.g. predefined priorities as follows

$$v_1 > v_5, \ v_5 > v_9, \ v_9 > v_{13}, \ v_{13} > v_1 \qquad (29)$$

It means that the particular priorities (29) can be expressed in the matrix form $\boxed{\mathbf{L}_v.\mathbf{v} \leq \mathbf{b}}$, where $\mathbf{b} = \mathbf{0}$ and

$$\mathbf{L}_v = \begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (30)$$

Considering $\mathbf{v}_0 = \mathbf{0}$ the incidence matrices and the initial state of the additional supervisor are the following

$$
\mathbf{F}_s = \begin{pmatrix} 100000000000000000 \\ 000010000000000000 \\ 000010000000000000 \\ 000000001000000000 \\ 000000001000000000 \\ 000000000000100000 \\ 000000000000100000 \\ 100000000000000000 \end{pmatrix} \quad \mathbf{G}_s^T = \begin{pmatrix} 000010000000000000 \\ 100000000000000000 \\ 000000001000000000 \\ 000010000000000000 \\ 000000000000100000 \\ 000000001000000000 \\ 100000000000000000 \\ 000000000000100000 \end{pmatrix}
$$

$$
{}^{s}\mathbf{x}_0^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}
$$

Although the problem of priorities was resolved, the number of the states of the system supervised by this supervisor is the same - i.e. 793.

Supervision in P/T PN models cannot compensate the deficiency of the presence of time.

In order to obtain the TPN model, let us assign the time specifications into the transitions of the P/T PN model.

Because the external priorities are not necessary here (namely, in our case they will be determined continuously with respect to the system evolution in the dependency on the current time) we will use the model having only the first supervisor proposed by (22)-(25).

Let us investigate the non-deterministic case, namely the uniform probability distribution of timing the transitions.

Consider the parameters

$$\mathbf{a} = \begin{pmatrix} 1.45, 4.5, 6.5, 1.45, 1.45, 4.5, 7.5, 1.45, 1.45, \\ 4.5, 8.5, 1.45, 1.45, 4.5, 9.5, 1.45, 0.95, 0.95 \end{pmatrix} \quad (31)$$

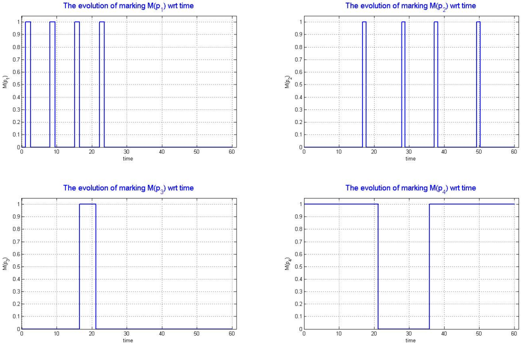
$$\mathbf{b} = \begin{pmatrix} 1.55, 5.5, 7.5, 1.55, 1.55, 5.5, 8.5, 1.55, 1.55, \\ 5.5, 9.5, 1.55, 1.55, 5.5, 10.5, 1.55, 1.05, 1.05 \end{pmatrix} \quad (32)$$

The particular entries a_i, b_i, $i = 1, 2, \ldots, 18$, of the vectors **a**, **b** correspond consecutively to the parameters belonging to the particular transitions t_i, $i = 1, 2, \ldots, 18$.

The simulation was performed on the time interval $\langle 0, 60 \rangle$.

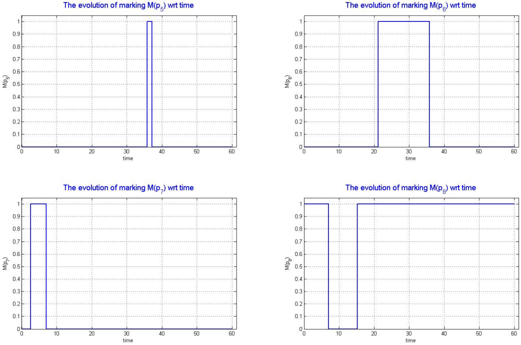
As the results of the simulation the development of marking in time can be found for all TPN places $p_1 - p_{20}$.

The TPN places p_{21} and p_{22} represent the supervisor. The simulation results are given in the Fig. 17.



... Case Study 5

... Performance Evaluation by Means of TPN-Based Model



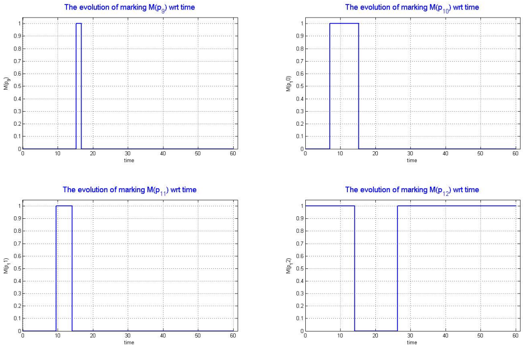
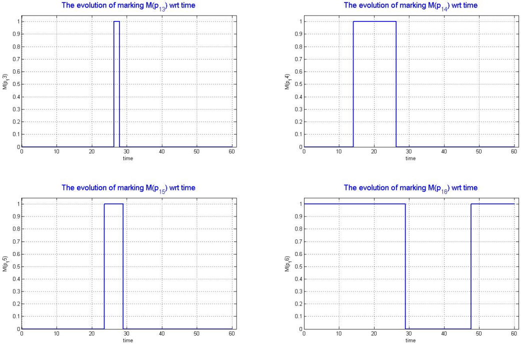


Figure 17: The simulation results at the discrete uniform probability distribution of timing. In the left column the marking development of the places p_1, p_3, p_5, p_7, p_9, p_{11} on the time interval $< 0, 60 >$ is displayed while in the right column the marking development of the places p_2, p_4, p_6, p_8, p_{10}, p_{12} on the same time interval is displayed.

... Case Study 5

... Performance Evaluation by Means of TPN-Based Model



... Case Study 5

... Performance Evaluation by Means of TPN-Based Model

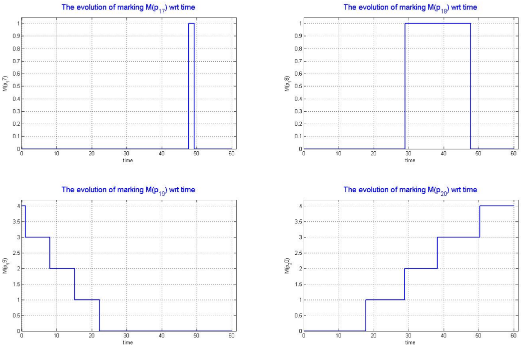


Figure 18: The simulation results at the discrete uniform probability distribution of timing. In the left column the marking development of the places p_{13}, p_{15}, p_{17}, p_{19} on the time interval $< 0, 60 >$ is displayed while in the right column the marking development of the places p_{14}, p_{16}, p_{18}, p_{20} on the same time interval is displayed.

8. Conclusion

The main idea of this paper is to point out the possibility of utilizing different kinds of Petri nets (P/T PN, TPN and FOHPN) at analytical approaches to modelling, analysing, control synthesis and performance evaluation of industrial processes.

Five case studies were presented and studied here:

In the Case Study 1 the P/T PN based approach to the DES supervisor synthesis was introduced.

In the Case Study 2 the TPN based approach to performance evaluation of relatively complex industrial process was presented.

In the Case Study 3 the FOHPN model of a hybrid system representing the industrial process was introduced.

In the Case Study 4 the FOHPN Model of a Batch Process was introduced and studied.

In the Case Study 5 the P/T PN model of ensuring the collision avoiding of 4 robots.

... Conclusion

In the Case Study 1 the algebraical approach to the supervisor synthesis based on the P/T PN P-invariants was introduced

In the Case Studies 2-5 the simulations of industrial processes in the tool Matlab by means of the tool HYPENS were performed.

Simulation results were introduced and described.

It can be said that Petri nets in general are very useful tool for modelling, analysing, performance evaluation and control of DES and HS in different branches of practice.

Thank you very much
for your attention!!!